Why Quantum Techniques Are a Good First Approximation to Social and Economic Phenomena, and What Next

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1. In general, different levels are described by different equations

- Many processes in our world occur at different scales:
  - cosmological processes describe what is happening on the level of Universe as a whole,
  - astrophysics describes what is happening on the level of Galaxies and stars,
  - Earth sciences describe what is happening on the level of a planet,
  - macrophysics, biology, and social sciences describes what is happening to our-size ("macro") level, and
  - finally, microphysics – mostly quantum physics – describes what is happening on the level of molecules, atoms, and elementary particles.
- Of course, there are some similarities between different levels – after all, all theses processes obey general laws of physics.

2. Different levels (cont-d)

- Still, the differences between different levels are usually much larger than these similarities.
- As a result, different level usually use different techniques, different methodologies.
- When people naively try to apply equations and ideas from one level to other levels, they rarely succeed.
- Naive 19 century attempts to describe electrons orbiting nuclei in a similar way as planets orbiting Earth led to paradoxes.
- For example, that, due to tidal forces (enhanced by electric charges), all electrons will fall on their nuclei after a few seconds.
- So, quantum physics was invented to avoid this paradox.

3. Different levels (cont-d)

- Similarly, attempts to naively apply Newtonian physics to the world as a whole lead to paradoxes – e.g.:
  - if we assume that the stars are uniformly distributed in the Universe,
  - then the overall intensity of light from the stars would be very large, and at night, it would be as light as in daytime.
- So, General Relativity was invented to avoid such paradoxes.
- Even for different aspects of the same level, naive transitions rarely work; e.g.:
  - while Darwinism is a great way to describe evolution of species,
  - attempts of Social Darwinism to explain social behavior the same way were not very successful.

4. But there is an important exception

- Interestingly, there is an important case when, unexpectedly:
  - equations that describe phenomena on one level
  - are strangely successful in a completely different level.
- This case is *quantum economics*, application of quantum physics to economic and social phenomena.
- This success is even more puzzling if we take into account that – as we have mentioned earlier:
  - attempts to use even closer-in-level phenomena like biological
  - were much less successful.

5. We need a quantitative explanation for this important exception

- It is easy to come up with *qualitative* explanations for the success of quantum methods in social studies.
- For example, when we study a social phenomenon, this study often changes the phenomenon itself.
- E.g., it is known that the very fact that a patient is visited by a doctor and has some tests done already makes many patients feel better.
- Of all other phenomena, only quantum processes have the same feature – that a measurement changes the state.
- So, it is reasonable to expect some analogies between social and quantum phenomena; however:
  - it is not just qualitative quantum ideas like this which are successful in studying social phenomena,
  - it is also quantitative quantum equations which are successful.

6. We need an explanation for this exception (cont-d)

- Why quantum methods are quantitatively successful in describing economic phenomena is a challenge.
- In this talk, we provide several explanations for this seemingly strange success.
- These explanations will hopefully make this success less puzzling.

7. Where can an explanation come from: general analysis

- In general, we can distinguish between three aspects of a physical theory:
- First, there is a mathematical aspect: equations that the real-world phenomena satisfy.
- For Newtonian physics, there are Newton's equations.
- For quantum physics, there are Schrödinger's equations.
- For General Relativity, there are Einstein's equations.
- However, equations are not all.
- To be useful, we must have techniques for solving these equations.
- Coming up with such techniques is usually as difficult (or even more difficult) than coming up with equations themselves.

8. Where can an explanation come from (cont-d)

- Newton would not have been very famous if all he did was:
  - write a system of differential equations describing how the planets move, and then
  - wait several centuries until computers would appear that could solve this system.
- Einstein would not have been that famous if all he did was:
  - write down a system of complex partial differential equations describing space-time geometry,
  - with no clue on how to solve them and how to compare his predictions with observations.
- This actually almost happened.
- David Hilbert, the leading mathematician of his time, independently discovered the same equations.
- He submitted his paper two weeks after Einstein.

9. Where can an explanation come from (cont-d)

- If he submitted it two weeks before – would we value Einstein's contribution at all?
- Actually, yes: all Hilbert did was came us with equations.
- On the other hand, Einstein also proposed some solutions – and a way to experimentally test these equations.
- This, in a few years led to a spectacular success.
- Finally, for the theory to become widely used, it is not enough to just have techniques for solving the corresponding equations.
- This would have limited this theory's use to academe where we have enough researchers and graduate students to apply these techniques.
- We cannot hire a PhD student for every single practical problem.
- To be practically useful, we need to have a large corpus of already solved problems that practitioners can use.

10. Where can an explanation come from (cont-d)

- For example, all cell phones take relativistic effects into account when dealing with GPS signals.
- However, the cell phone company does not have to hire a physicist every time a new model of a cell phone is designed.
- They can use known solutions.
- This applies to all physical theories.
- This applies, in particular, to quantum physics:
  - there are equations,
  - there are techniques for solving these equations, and
  - there are numerous solutions of these equations.

11. Where can an explanation come from (cont-d)

- We will show that:
  - each of the three aspects of quantum physics provides some explanation of
  - why quantum equations can be successfully applied to social phenomena.
- Taken together, all three explanations form a reasonably convincing case.
- So let us consider these aspects one by one.

12. First explanation: quantum formulas provide a good description for many phenomena in general

- First, let us consider the mathematical aspect of quantum physics – the corresponding mathematical equations.
- We will show that the mathematical formulas of quantum physics provide:
  - a good approximate description for many phenomena,
  - not only phenomena from micro-world and from economics.

13. Towards a general description of real-life phenomena

- In most real-life situations, we have many objects of similar type:
  - a galaxy consists of many stars,
  - a species consists of many individuals,
  - a macro-object consists of many molecules,
  - a country or a firm is formed by many people, etc.
- Each of these objects is characterized by the values of several quantities.
- The more quantities we study, the more accurate picture of this object we get.
- The number of objects is usually very large, so it is not realistic to keep track of all these objects.

14. A general description of real-life phenomena (cont-d)

- A more realistic idea is to keep track of the corresponding distributions:
  - what is the proportion of stars of given brightness,
  - what is the proportion of employees whose salary is within a given range, etc.
- Most practical situations are complex, each quantity is determined by many independent factors.
- For example, in a big multi-national corporation, a person's salary:
  - depends on the person's skills,
  - depends on the number of years with the company,
  - depends on the geographic location – employees located in more expensive-to-live areas usually get higher salary – etc.

15. A general description of real-life phenomena (cont-d)

- It is known that the distribution of a joint effect of a large number independent factors is close to Gaussian.
- This follows from the Central Limit Theorem, according to which:
  - when the number of relatively small independent random variables increases,
  - the distribution of their sum tends to Gaussian.
- So, we can conclude that the joint distribution of quantities $v_1, \ldots, v_n$ characterizing individual objects is (close to) Gaussian.

16. Need for an approximation

- In general, a multi-D Gaussian distribution is uniquely determined by its first two moments, i.e.:
  - by its means $m_i \stackrel{\text{def}}{=} E[v_i]$ and
  - by its covariance matrix $C_{ij} \stackrel{\text{def}}{=} E[(v_i - m_i) \cdot (v_j - m_j)]$.
- So, to describe the distribution, we need:
  - to know n values of the means and
  - to know $\dfrac{n \cdot (n+1)}{2}$ values of the symmetric matrix C_{ij},
  - the total of $V = n + \dfrac{n \cdot (n+1)}{2}$ parameters.
- These values need to be determined experimentally, and herein lies a problem.
- In general, according to statistics, based on N observations, we can estimate the value of a parameter with relative accuracy $\varepsilon \approx 1/\sqrt{N}$.

17. Need for an approximation (cont-d)

- So, to find the value of a parameter with given relative accuracy $\varepsilon > 0$, we need to perform $N(\varepsilon) \approx \varepsilon^{-2}$ observations.
- To find the values of V parameters, we therefore need to perform $V \cdot N(\varepsilon) \approx V \cdot \varepsilon^{-2}$.
- For large n, this becomes too large; for example:
  - if we are interested in comparing countries, and we want to characterize even $n = 3$ quantities with accuracy $\varepsilon \approx 20\%$,
  - then we need a sample of 225 countries.
- However, there are not that many countries in the world.
- To be more precise, means m_i are not a problem, we can determine them.
- The problem is to determine the elements of the covariance matrix.

18. A general description of real-life phenomena (cont-d)

- This simple argument shows that often, we cannot experimentally determine the actual Gaussian distribution.
- Indeed, this distribution depends on too many parameters.
- We therefore need to find a lower-parametric family of distributions that we will use for an approximate description of the phenomena.

19. How can we find such an approximation?

- How can we find a natural, intuitively clear approximation?
- Most of us do not have a good intuition about probability distributions, but we do have a good intuition about geometric descriptions.
- Good news is that there is a natural geometric description of a multi-D Gaussian distribution.
- Namely, it is known that:
  - we can represent the components $\Delta v_i = v_i - m_i$ of a multi-D Gaussian distribution with 0 means ($E[\Delta v_i] = 0$)
  - as linear combinations of standard independent Gaussian random variables $\xi_1, \ldots, \xi_n$ for which $E[\xi_k] = 0$, $E[\xi_k^2] = 1$:

$$\Delta v_i = \sum_{k=1}^{n} v_{ik} \cdot \xi_k.$$

20. How can we find such an approximation (cont-d)

- In this representation, the covariance $E[\Delta v_i \cdot \Delta v_j]$ takes the form
$$E[\Delta v_i \cdot \Delta v_j] = \sum_{k=1}^{n} v_{ik} \cdot v_{jk}.$$
- This is exactly the formula for the dot (scalar) product of the two n-dimensional vectors.
- So, we conclude that:
  - each difference each difference Δv_i is represented by an n-dimensional vector $\vec{v}_i = (v_{i1}, \ldots, v_{in})$, and
  - the covariance is equal to the dot products of these vectors:
$$E[\Delta v_i \cdot \Delta v_j] = \vec{v}_i \cdot \vec{v}_j.$$
- In particular, the variance $V[\Delta v_i] \stackrel{\text{def}}{=} E[(\Delta v_i)^2]$ has the form $V[\Delta v_i] = (\vec{v}_i)^2 = \|\vec{v}_i\|^2$, where $\|\vec{a}\|$ denotes the length of the vector $\vec{a}$.
- This provides an exact n-dimensional representation of the situation.

21. How can we find such an approximation (cont-d)

- As we have mentioned, we often do not have enough experimental data to determine this exact n-dimensional representation.
- So, a natural idea is to have a lower-dimensional approximation.
- This is indeed natural; for example:
  - when we do not have enough data to find a full 3D picture of some object,
  - we often have enough data to determine its 2-D projection.
- In other words:
  - instead of the original (ideal) multi-D vectors $\vec{v}_i$,
  - we use lower-dimensional approximate vectors $\vec{V}_i = (V_{i1}, \ldots, V_{id})$, for $d \ll n$.
- We try to find the vectors $\vec{V}_i$ so that the corresponding dot products are close to the ideal ones: $\vec{V}_i \cdot \vec{V}_j \approx \vec{v}_i \cdot \vec{v}_j$.

22. The simplest such approximation leads, in effect, to a quantum description

- The most intuitively clear pictures are 2-D ones, with $d = 2$.
- In this case, each quantity v_j is represented as by a 2-D vector $\vec{V}_i = (V_{i1}, V_{i2})$, and its variance is approximately equal to $\|\vec{V}_i\|^2 = V_{i1}^2 + V_{i2}^2$.
- How is all this related to complex numbers – one of the main techniques of quantum physics?
- The relation is straightforward: there is a natural geometric representation of complex numbers, where:
  - each number $a + b \cdot \mathrm{i}$ (and $\mathrm{i} \stackrel{\text{def}}{=} \sqrt{-1}$)
  - is represented by a point (a, b) on a plane.
- In this representation:
  - the absolute value $|a + b \cdot \mathrm{i}|$ of the complex number
  - is equal to the length $\|(a, b)\| = \sqrt{a^2 + b^2}$ of the corresponding vector (a, b).

23. The simplest such approximation leads, in effect, to a quantum description (cont-d)

- Thus, each vector $\vec{V}_i = (V_{i1}, V_{i2})$ characterising a quantity v_i can be naturally represented as a complex number $V_{i1} + V_{i2} \cdot \mathrm{i}$.
- The standard deviation of the quantity v_i is equal to the absolute value of this complex number.
- So, in this approximation, indeed we naturally get a quantum-like description of general objects.

24. Second explanation: quantum formulas are the computationally fastest way to describe nonlinear phenomena

- Now let us concentrate:
  - on the computational, algorithmic aspect of quantum physics, and
  - on why namely the computational aspects of quantum physics are helpful in describing social phenomena,
  - and not of any other area of physics.
- To provide this explanation, let us consider the computational aspects of physics from the most general viewpoint.
- From the purely theoretical viewpoint, once we have an equation, we can find its solution.
- Worse comes to worse, we can try all possible values, all possible combinations of values, etc.

25. Second explanation (cont-d)

- Of course, on the abstract level, there are infinitely many possible values of each physical quantity.
- And it is not possible to try all infinitely many values.
- However, in practice, since measurements are never absolutely accurate, we cannot distinguish between close values; for example:
  - if our measuring instrument only works on the range from 0 to 10, and
  - measures the value of the corresponding quantity with accuracy 0.1,
  - then we can, in effect, only have values 0, 0.1, 0.2, . . . , 9.9, 10 – any other value will be indistinguishable from one of these.
- Of course, trying all possible combinations is not a practically feasible approach.

26. Second explanation (cont-d)

- For many unknowns, it can take an astronomical amount of time, up to the time larger than the lifetime of the Universe.
- So, from the practical viewpoint, the question is not to find *an* algorithm, the problem is to find a *reasonably fast* algorithm.
- This is a big challenge.
- For example, we have learned to predict tomorrow's weather reasonably well – it takes several hours on a high-performance computer.
- Almost the same algorithms can, in principle, predict where a devastating tornado will turn in the next 15 minutes.
- Unfortunately, this prediction would also take several hours, which makes it useless.
- From this viewpoint, an important question is how to perform computations as fast as possible.

27. Second explanation (cont-d)

- Computations consist of several elementary steps, steps on which we compute some elementary functions.
- For example, current computers use min, max, addition, and multiplication as such steps.
- Everything else is implemented as a sequence of these operations:
  - whether it is computing the inverse $1/a$
  - or computing the value of the sine function.
- So:
  - to make computations faster,
  - it is important to select elementary computational steps – i.e., corresponding functions – which are the fastest to compute.
- Which functions are the fastest to compute?
- The first idea is that functions are either linear or non-linear.

28. Second explanation (cont-d)

- Of course, linear functions are faster to compute than nonlinear ones.
- So we should use computing linear functions as elementary computational operations.
- We cannot limit ourselves to linear functions only:
  - if we only perform linear combinations, then, since the composition of linear function is linear, we will only get linear functions,
  - but many real-life processes are nonlinear.
- So, some nonlinear elementary operations are needed.
- However, since we want to speed up overall computations, we should have as few non-linear elements as possible.
- So, we end up with a computational scheme in which most operations are linear, but sometimes some nonlinear operations are needed.
- Which non-linear operations should we use?

29. Second explanation (cont-d)

- To predict the results of deterministic processes, a natural idea is to use some nonlinear functions.
- So, we end up with a computation scheme in which we interchangingly apply linear transformations and some simple nonlinear ones.
- This is exactly what neural networks are doing – at each layer, we:
  - first form a linear combination $s = w_0 + w_1 \cdot s_1 + \ldots + w_n \cdot s_n$ of signals coming from the previous layer, and
  - then, we apply some nonlinear function $F(x)$ (known as *activation function*) to this result s, returning the value $F(s)$.
- Many processes, however, are nondeterministic, in the sense that:
  - based on the available information,
  - we can only predict the results with some probabilities.
- In this case, it makes sense to have non-deterministic nonlinear components, that return different results with different probabilities.

30. Second explanation (cont-d)

- This is exactly what is happening according to the quantum physics, where interchanhingly, we have:
  - linear transformations – which correspond to normal dynamics as described by Schrödinger's equation – and
  - measurement process – in which we get different results with different probabilities.
- So, a quantum-type description is the most computationally efficient way to describe nondeterministic phenomena.
- And social phenomena are, of course, largely nondeterministic.
- So, from the computational viewpoint, we also naturally arrive at a quantum-type description of social phenomena.

31. Third explanation: quantum physics has many solved problems

- Finally, let us take into account that in quantum physics, we have a large corpus of solved complex problems.
- Why is this important?
- It is known that all classes of sufficiently complex problems (they are known as *NP-hard*) can be reduced to each other.
- Thus:
  - once we have come up with an efficient method of solving complex problems from one application area,
  - we can reduce problems from other areas to these problems and
  - thus, get a good algorithm for solving problems from other areas as well.

32. Third explanation (cont-d)

- So:
  - a natural way to solve complex problems in economics and finance
  - is to reduce them to complex problems in other application areas,
  - to problems for which solutions are mostly known.
- One such area is physics – analysis of the physical world.
- Its equations are often very complex.
- Still, during several centuries of physics, researchers have found reasonably efficient algorithms for solving many of these equations.
- Thus, to solve complex problems in economics and finance, it makes sense to reduce them to solvable complex problems from physics.

33. Third explanation (cont-d)

- It is known that to get an adequate description of a physical phenomenon:
  - it is necessary to take quantum effects into account,
  - i.e., to get into the domain of quantum physics.
- Because of this, the most complex physical equations are equations of quantum physics.
- Thus, a natural idea is to reduce complex equations arising in economics and finance:
  - to complex quantum equations,
  - namely, to complex quantum equations for solving which we have efficient algorithms.

34. Third explanation (cont-d)

- Then:
  - by solving the corresponding quantum equation and translating the solutions back into the language of economics and finance,
  - we can get efficient algorithms for solving complex economic and financial problems.
- This is exactly what quantum econometrics is doing!
- So, we have yet another natural explanation of the empirical success of quantum techniques in economics and in social sciences in general.

35. Beyond quantum

- In the previous part, we explained why quantum techniques work reasonably well in economics.
- Of course, economics is different from quantum world.
- So quantum equations provide only a first approximation to economic situations.
- A natural question is: how can we get an even more accurate approximation?
- How can we modify quantum equations so that they will provide an even more adequate description of social phenomena?
- Based on the above explanations, we can come with some recommendations about these modifications.

36. Beyond quantum (cont-d)

- Indeed, we showed that a quantum-style complex-valued description of different objects naturally appears if:
  - as an approximation of the general geometric description – with n-dimensional vectors,
  - we consider 2-D vectors.
- Of course, this is just an approximation.
- To make this approximation more accurate, the next natural way is to approximate the objects by 3-D, 4-D, etc. vectors.
- We hope that:
  - this natural generalization of quantum-style techniques
  - will lead to a more accurate representation of social and economic phenomena.

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