# Why Decreased Gaps Between Brain Cells Cause Severe Headaches: A Symmetry-Based Geometric Explanation

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## 1. Empirical Fact

- It is well known that living creatures consist of cells.
- Cells is all you see when you look at any tissue under a microscope.
- This is how scientists understood the structure of the living creatures.
- However, starting with the 1960s, it was determined that in living creatures:
  - there is always a liquid-filled gap between cells,
  - a gap that closes when the matter is no longer alive.
- The gaps were first discovered when:
  - to preserve the structure as much as possible,
  - researchers instantaneously froze the cells culture.
- Since then, new techniques have been developed to study these gaps.

## 2. Empirical Fact (cont-d)

- The corresponding studies showed that these gaps play important biological roles.
- For example:
  - in abnormal situations, when the gaps between brain cells drastically decrease,
  - a patient often experiences several headaches and other undesired effects.

## 3. Why?

- As of now, there are no universally accepted explanations for this empirical fact.
- In this paper, we show that this empirical phenomenon naturally follows from symmetry-based geometric ideas.

## 4. Normal case: geometric description

- Let us first consider the case when there are gaps between cells.
- We want to know how cells interact with each other.
- All interactions are local; thus:
  - to analyze the interaction between the cells,
  - we need to consider a small area on the border between two neighboring cells.
- The border of each cell is usually smooth.
- Locally, each smooth surface can be well-approximated by its tangent plane.
- The smaller the area, the more accurate the approximation.
- Thus, with good accuracy, we can locally represent the border of each cell by a plane.
- The border of the neighboring cell is also represented by a plane.

## 5. Normal case: geometric description (cont-d)

- Since we consider the situation when there is a gap, the borders do not intersect.
- When the two planes do not intersect, this means that they are parallel to each other.
- Thus, the normal configuration can be described by two parallel planes corresponding to two neighboring cells.

## 6. Abnormal case: geometric description

- In the abnormal case, the gaps drastically decrease, to the extent these gaps become undetectable.
- Thus, with good accuracy, we can conclude that in this case, there is, in effect, no gaps between the two cell.
- Thus, both cells can be described by a single plane, the plane that serves as a common border of the two cells.

## 7. We want to study dynamics

- For a living creature:
  - there is a usually a stable state, and
  - then there are dynamic changes, when a change in one cell causes changes in others.
- To study dynamics, we therefore need to study how disturbances propagate.
- According to physics, all interactions are local.
- So, for a perturbation in one cell to reach another cell, this perturbation first need to reach the border of the original cell.
- The simplest perturbation is when the perturbation is located at a single point on the cell's border.

## 8. We want to study dynamics (cont-d)

- Every other perturbation can be viewed as a combination of such point-wise perturbations corresponding to all affected points.
- So, to study how general perturbations propagate, it is necessary to study how point-wise perturbations propagate.

## 9. Role of symmetries

- Most physical processes do not change if we apply:
  - shift,
  - rotation, or
  - scaling – i.e., replace, for some $\lambda > 0$. each point with coordinates $x = (x_1, x_2, x_3)$ with a point with coordinates $(\lambda \cdot x_1, \lambda \cdot x_2, \lambda \cdot x_3)$.
- Physical processes are invariant with respect to such geometric symmetries.
- Thus:
  - if we start with the initial configuration which is invariant with respect to some of these symmetries,
  - the resulting configuration will also be invariant with respect to the same geometric transformations.
- Let us analyze how this idea affects the propagation of perturbations between the cells.

## 10. Normal case: what are the symmetries and what are possible dynamics

- Let us first consider the normal case, when we have:
  - two parallel planes and
  - a point in one of these planes – the location of the original perturbation.
- One can see that out of all above-listed geometric symmetries:
  - the only symmetries that keep this configuration invariant are rotations around the fixed point in the first plane,
  - i.e., in 3D terms, rotations around the axis that goes through this point orthogonally to the plane
  - (and, since the planes are parallel, orthogonally to both planes).
- The initial configuration has this symmetry.
- So, the resulting configuration – observed after some time – should also has the same symmetry.

## 11. Normal case: what are the symmetries and what are possible dynamics (cont-d)

- In particular, with respect to what perturbations we can have on the other plane – i.e., on the border of the neighboring cell:
  - we can have a single point (on the same axis),
  - or we can have a rotation-invariant planar region (e.g., a disk centered at this point) that reflects inevitable diffusion.
- These cases correspond to usual information transfer between the cells.
- One of the possibilities is that the resulting configuration will involve all the points of the second plane.
- However, this is not the only configuration resulting from diffusion.

## 12. Abnormal case: what are the symmetries and what are possible dynamics

- Let us now consider the abnormal no-gaps case, when we have:
  - a single plane – a joint boundary between the cells, and
  - a point in this plane, which is the location of the original perturbation.
- In this case, in addition to rotations, the corresponding configuration has an additional symmetry – scalings around the given point.
- Thus, the resulting configuration should be invariant not only with respect to the rotations, but also with respect to these scalings.
- Due to inevitable diffusion, we expect the plane part of the resulting configuration to include more than a single point.

## 13. Abnormal case: what are the symmetries and what are possible dynamics (cont-d)

- However, one can easily see that:
  - every two points on a plane – which are both different from the original point
  - can be obtained from each other by an appropriate rotation and scaling.
- Thus:
  - once the resulting rotation- and scale-invariant configuration contains at least one point which different from the original point,
  - it will automatically include all the points in the plane.
- In other words, in the presence of even small diffusion, a local point-wise perturbation will lead to a perturbation of the whole boundary.

## 14. Abnormal case: what are the symmetries and what are possible dynamics (cont-d)

- This perturbation will spread to other cells – and cause a global all-cells-involving perturbation.
- This is exactly what corresponds to a severe headache, when many cells are affected.

## 15. Summarizing: this explains the observed phenomenon

- Our analysis has shown the following.
  - In the normal case – when there are gaps between brain cells – while we can have global-brain effects like severe headache, this is not inevitable: we can also have a usual information transfer between cells.
  - On the other hand, in the no-gaps case, effects like severe headache are inevitable.
- Thus:
  - the thinner the gaps, the closer the resulting configuration is to the no-gaps case,
  - the more probable it is that severe headaches (and other global effects) will occur.
- This is exactly what is observed.
- So, our symmetry-based geometric analysis indeed explains the observed phenomenon.

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