# Towards a Psychologically Natural Relation Between Colors and Fuzzy Degrees

Victor L. Timchenko[1], Yuriy P. Kondratenko[2,3], Olga Kosheleva[4], Vladik Kreinovich[4], and Nguyen Hoang Phuong[5]

[1]Admiral Makarov National University of Shipbuilding
Mykolaiv 054025, Ukraine, vl.timchenko58@gmail.com
[2]Petro Mohyla Black Sea National UNiversity
Mykolaiv 054003, Ukraine
[3]Institute of Artificial Intelligence Problems
Ministry of Education and Science (MES) of Ukraine and
National Academy of Sciences (NAS) of Ukraine Kyiv 01001, Ukraine
y_kondrat2002@yahoo.com
[4]University of Texas at El Paso, El Paso, Texas 79968, USA
olgak@utep.edu, vladik@utep.edu
[5]Artificial Intelligence Division, Information Technology Faculty
Thang Long University, Nghiem Xuan Yem Road
Hoang Mai District, Hanoi, Vietnam, nhphuong2008@gmail.com

# 1. Need for processing fuzzy information

- In many real-life situations, be it driving a car or treating a patient:
  - humans provide a good quality control,
  - often control which is of better quality than what current automatic controllers can provide.
- It is therefore desirable:
  - to incorporate the knowledge and experience of these human controllers into the automatic systems,
  - to make these systems more effective.
- Such incorporation is not easy.
- One of the main reasons why it is not easy is that human experts often describe their control strategies:
  - not in precise easy-to-program mathematical form,
  - but by using imprecise ("fuzzy") words from natural language such as "small".

## 2. Need for processing fuzzy information (cont-d)

- We need to transform this knowledge into precise computer-understandable terms.

- For this purpose, Lotfi Zadeh invented a special technique that he called *fuzzy*.

- In this technique, to describe an imprecise property like "small", we ask the expert to assign:

  - to each possible value $x$ of the corresponding quantity,
  - a degree – from the interval $[0, 1]$ – describing to what extend this value has the given property
  - e.g., is small.

# 3.  Need for discrete fuzzy degrees

- From the purely mathematical viewpoint, there are infinitely many degrees in the interval $[0, 1]$.

- So nothing should prevent experts to assign degrees like 0.2134 or 0.6778.

- However, in practice, people cannot meaningfully distinguish between many degrees.

- According to psychology, they can distinguish between 7 plus minus 2 different degrees.

- For most people, this number is 7, so it makes sense to use seven different degrees.

- We have an absolute "yes" and an absolute "no" situations.

- It is also reasonable to have a neutral situation:
  - when we have exactly as many arguments for "yes"
  - as for "no".

## 4.   Need for discrete fuzzy degrees (cont-d)

- To make 7, we should have four degrees corresponding to some degree of confidence in a statement or it is negation.

- Thus, we arrive at the following natural list of degrees:

| no | very probably no | probably no | neutral | probably yes | very probably yes | yes |
|----|------------------|-------------|---------|--------------|-------------------|-----|

## 5. Need for fast computations

- In many practical problems, it is desirable to perform computations as fast as possible.

- This is especially true in problems related to real-time control,

# 6. Need for optical computing

- One of the factors that limits the computation speed is the speed with which signals pass:
  - from one part of the processing unit
  - to another.

- It is therefore desirable to make sure that:
  - this signal transfer occurs
  - at the highest possible speed.

- According to physics, the fastest possible speed is the speed of light.

- Thus, to speed up computations, it is desirable to use, in computations:
  - electromagnetic waves
  - in particular, visible light.

- Such computations are known as *optical computing*.

# 7. Need for color optical computing

- It is well known that every optical signal can be represented as a combination of several signals of pure color.

- This fact that discovered by Newton, who showed that:

  - each light can be decomposed into pure colors,
  - and then these pure colors can be assembled back, into the original signal.

- Thus, a natural idea is to use signals of different color.

- Such computations are known as *color optical computing*.

- In particular, it is desirable to use color optical computing to process fuzzy data.

## 8. Historical comment

- From the mathematical and signal processing viewpoints:
  - Newton's discovery about colors
  - was the first empirical proof of what is now known as Fourier techniques.
- Their main idea is that each signal can be presented as a combination of sinusoids with different frequency.
- And pure colors are exactly signals of fixed frequency.

## 9.   Problem: it is desirable to have a natural relation between colors and fuzzy degrees

- We want to use color optical computing for processing fuzzy data.

- So, we need to assign different colors to different fuzzy degrees.

- Fuzzy techniques process intuitively clear notions like "small".

- It would be advantageous:

    - to extend this naturalness to data processing as well,
    - i.e., to make the relation between colors and fuzzy degrees natural.

- In this talk, we show how to come up with such a natural relation between colors and fuzzy degrees.

# 10. What colors should we consider?

- It is known that in our color perception, we have three basic colors – red (R), blue (B), and green (G).

- From our viewpoint, every other color can be represented as a combination of these three basic colors:

  - a combination of red and green (R + G) leads to yellow (Y),
  - a combination of red and blue (R + B) leads to magenta (M),
  - a combination of green and blue (G + B) leads to cyan (C), and
  - a combination of all three colors (R + G + B) leads to white (W).

- Also, the absence of colors corresponds to black (Blc).

- These are the colors that we need to assign to different fuzzy degrees.

# 11.   What is a natural analog of "neutral"?

- The most easy thing is to assign a color to the degree "neural".

- Indeed, in this degree, the expert has exactly as many argument for as arguments again.

- In other words, in such situation, the expert cannot decide, i.e., in effect, has no contribution to the decision process.

- It is therefore natural to associate this no-information degree with no-information color – i.e., with the color Black.

- What remains to be done is to associate appropriate colors with the remaining six degrees:

| no | very probably no | probably no | probably yes | very probably yes | yes |
|----|------------------|-------------|--------------|-------------------|-----|
| ?  | ?                | ?           | ?            | ?                 | ?   |

# 12. What is known

- Psychologists have studied the relation between colors and decision making.

- They found an association of some colors with decisions.

- One has to be cautious since this research is mostly based on US customers.

- However, still, these associations are largely universal.

- Let us sort this information in the order in which these colors appear in the spectrum:

- The color Yellow is associated with Inexpensive and Low Quality.

- The color Green is associated with Good Taste.

- The color Blue is associated with High Quality and Reliability.

- Also, the color Black is associated with Expensive.

- How can we translate these associations into fuzzy degrees?

## 13.   What is known (cont-d)

- Let us start with Yellow.

- Low Quality is not good.

- However, because of Inexpensive, it is not a clear No: sometimes, we buy low quality things because we cannot afford things of better quality.

- We are not happy when we do it.

- So a natural idea is to assign, to this color, the degree "very probably no".

- Good Taste is a positive characteristic, but it is only one – and often, not the most important characteristic.

- More important is how usable it is, taste is secondary.

- It is therefore natural to associate, to this color, the degree "probably yes".

## 14.   What is known (cont-d)

- Blue has only positive characteristics, both are important, so let us associate it with "yes".

- What about Black?

- The fact that something is Expensive does not mean that we should buy it – only snobs buy things just because they are expensive.

- It also does not mean that we should *not* buy it: expensive is a price that we pay for high quality.

- In other words, from the viewpoint of decision making:

    – the very fact that something is expensive
    – does not help to decide whether we but it or not, it is neutral.

- This is in perfect agreement with our previous association of Black with "neutral".

- Let us summarize the above arguments by placing the corresponding colors into our table:

| no | very probably no | probably no | probably yes | very probably yes | yes |
|---|---|---|---|---|---|
| ? | R + G | ? | G | ? | B |

# 16. How to extend this assignment to other colors: a natural idea

- We need to extend the above psychologically natural relation between colors and fuzzy degrees to the remaining degrees.

- A natural idea is that:
    - the degree corresponding to the mixture of two colors
    - should be located between the degrees corresponding to these two colors.

- So, we come up with the following conclusions.

- According to the above idea, Y = R + G should be between R and G.

- In our table, the degree corresponding to G is to the right of the degree corresponding to Y.

- So, the degree corresponding to R should be to the left of the degree "most probably no" corresponding to Y.

## 17. How to extend this assignment to other colors: a natural idea (cont-d)

- The only degree which is to the left of "very probably no" is "no".
- So, we assign, to Red, the degree "no":

| no | very probably no | probably no | probably yes | very probably yes | yes |
|----|------------------|-------------|--------------|-------------------|-----|
| R  | R + G            | ?           | G            | ?                 | B   |

- Similar, the degree corresponding to G + B should be:
  - in between the degrees corresponding to G and B,
  - i.e., in between the degrees "probably yes" and "yes".
- There is one degree in between: "very probably yes".
- So, we assign this degree to the color G + B.

| no | very probably no | probably no | probably yes | very probably yes | yes |
|----|------------------|-------------|--------------|-------------------|-----|
| R  | R + G            | ?           | G            | G + B             | B   |

## 18. How to extend this assignment to other colors: a natural idea (cont-d)

- The only remained un-assigned degree is "probably no".

- The only remaining un-assigned color is R + B.

- So we naturally match them.

- This matching, by the way:
    - is in perfect accordance with our idea,
    - since this degree is in between degrees "no" and "yes" corresponding to Red and Blue.

- Thus, we arrive at the following conclusion.

# 19. Conclusion

- The following is the natural association of colors and fuzzy degrees:

| no | very probably no | probably no | probably yes | very probably yes | yes |
|----|------------------|-------------|--------------|-------------------|-----|
| R | R + G | R + B | G | G + B | B |

- To make this relation clearer, let us reformulate it with full names of the resulting colors:

| color | structure | degree |
|-------|-----------|--------|
| Red (R) | R | no |
| Yellow (Y) | R + G | most probably no |
| Magenta (M) | R + B | probably no |
| Black | | neutral |
| Green (G) | G | very probably yes |
| Cyan (C) | G + B | probably yes |
| Blue (B) | B | yes |

## 20. Can we make this natural relation less subjective?

- Our relation relies on the results of empirical psychological studies.

- A natural question is: can we make our conclusions:

  - less subjective, based only on some objective characteristics,
  - so that we will not need to rely on these empirical results?

- Let us try to do it.

- As we have mentioned, a color is described by its frequency.

- We are talking about using color to communicate.

- From this viewpoint, the higher the frequency, the most information we can transfer by using signals of this color.

- So, the larger the frequency, the more positive our attitude to different colors.

- Let us show how this idea helps.

- Among the three basic colors Red, Green, and Blue:

  – Red corresponds to the smallest frequency,

  – Green to the intermediate one, and

  – Blue to the largest one.

- So we have: $R < G < B$.

- Let us extend this order to combinations of basic colors.

- It is natural to assume – as we did before – that:

  – the quality of a combination of two colors

  – should be in between the corresponding colors.

- So, we get $R < R + G < G < G + B < B$ and

$$R < R + B < B.$$

- It is also reasonable to require that:
  - if we replace one color in a combination with a better color,
  - the quality of the combination improves.
- As a result, we get $R + G < R + B < G + B$.
- By combining the above comparisons, we end up with the following order:

$$R < R + G < G, R + B < G + B < B.$$

- This is *almost* what we got by using the empirical data, but not exactly:
  - by using the general ideas,
  - we cannot decide which is better: Green (G) or Magenta $(M = R + B)$.
- It is easy to see why we cannot get the full order based only on the objective characteristics.

- Indeed, the original situation remains the same if we reverse the order, i.e., if we:

  - swap R and B, while keeping G intact, and
  - replace each < with >.

- One can easily see that the resulting relations do not change under this transformation.

- However, we cannot have R + B < G.

- Indeed:

  - this transformation will transform it into its opposite
    R + B > G, and
  - these two inequalities cannot be both true.

- Similarly, we cannot have $G < R + B$; indeed:

  – this transformation will transform it into its opposite $G > R + B$, and

  – these two inequalities cannot be both true.

- This may be related to fact that:

  – in contrast to Green and to many other colors,

  – magenta $(R + B)$ is *not* a real color,

  – in the sense that there is no frequency corresponding to magenta.

- The only way to get a perception of magenta is to mix several colors.

- This is different from other combinations, such as Yellow:

  - yes, in the Red-Green-Blue scheme, Yellow is a combination of two colors,

  - but there is also a wavelength range 565–590 THz corresponding to Yellow.

- Similarly, there is a wavelength range 600–620 THz corresponding to Cyan (which we represented as G + B).

# 26. But maybe we can get the full order this way?

- What if we take into account:
  - not only the *order* between the frequencies corresponding to three basic colors,
  - but also the *numerical ranges* of these frequencies.
- These frequencies are:
  - 440–480 THz for Red, centered at 440,
  - 530–600 THz for Green, centered at 565, and
  - 620–670 THz for Blue, centered at 645.
- We can see that the central point (565) of the Green frequency range:
  - is much further to the right
  - than the average $(440 + 645)/2 = 542.5$ of the central points of the frequency ranges corresponding to Red and Blue.
- So, it is not surprising that G feels better that R + G.

## 27. Acknowledgments