Why Green Wavelength Is Closer to Blue Than to Red and How It Is Related to Computations: Information-Based Explanation

Victor L. Timchenko[1], Yuriy P. Kondratenko[2], Olga Kosheleva[3], Vladik Kreinovich[3], and Nguyen Hoang Phuong[4]

[1]Admiral Makarov National University of Shipbuilding
Mykolaiv, Ukraine, vl.timchenko58@gmail.com
[2]Mohyla Black Sea National University, Mykolaiv, Ukraine, and
Institute of AI Problems, Kyiv, Ukraine, y_kondrat2002@yahoo.com
[3]University of Texas at El Paso, El Paso, Texas 79968, USA
olgak@utep.edu, vladik@utep.edu
[4]Thang Long University, Hanoi, Vietnam, nhphuong2008@gmail.com

1. What we have done so far

- In our previous papers, we have analyzed the idea of using light signals of three basic colors, red, green, and blue:
  - to speed up computations,
  - in particular, fuzzy-related computations.
- This idea has indeed led to successful applications.

2. A natural next question

- With any successful technique, a natural question is: can we do better?
- Can we change some parameters of the currently successful method and thus, make it even more successful?
- Or maybe:
  - the parameters that we used are already close to optimal, and
  - we cannot get a more effective method by simply chaging these parameters – for that, we need new ideas.

3. What are the parameters that we used?

- In our work:
  - out of all possible triples of colors,
  - we selected the triple of basic colors, i.e., red, green, and blue.
- The main reason why we selected the three basic colors is that they form the basis of our color perception.
- At first glance, this may sound like a naive explanation.
- However, as many based-on-biology explanations, it has depth behind it.
- Indeed, we, as biological creatures, are a product of billions of years of improving evolution.
- If some alternative features were better, they would have been implemented many millennia ago.
- From this biological perspective, most of our features are close to optimal.

4. What are the parameters that we used (cont-d)

- So it makes sense to copy these features.
- And indeed, many engineering designs copied from nature led to many successes.
- In computing, great examples of such successes are evolutionary computations and neural networks.

5. A deeper explanation is desirable

- Ok, but maybe the three basic colors:
  - are optimal for human visual perception
  - because of the specifics of the biological substances that can detect these colors.
- If this is the case, then this human optimality has nothing to do with our objective – to speed up computations.
- From the viewpoint of speeding up computations, it is desirable to come up with some more fundamental explanation.

6. A seemingly natural idea

- What if the current selection of three colors is not optimal for our purposes?
- Let us look at this selection from a fresh viewpoint.
- When we select three colors:
  - we do not want any two of them to be too close to each other;
  - in that case, it would be difficult to distinguish between the two close colors in real-life situations, when noise is present.
- It is thus reasonable to select colors that are as far away from each other as possible.
- In particular, once we selected the two colors with the smallest and largest wavelenghts $\underline{\lambda}$ and $\overline{\lambda}$, it makes sense:
  - to select, for the intermediate color,
  - the wavelength λ for which the largest of the two wavelength differences $\max(\lambda - \underline{\lambda}, \overline{\lambda} - \lambda)$ is as large as possible.

7. A seemingly natural idea (cont-d)

- One can easily see that this optimization leads to selecting the midpoint $\lambda = \dfrac{\underline{\lambda} + \overline{\lambda}}{2}$.
- This is the point which is equally close to both colors $\underline{\lambda}$ and $\overline{\lambda}$.
- However, in human perception, the intermediate color green is closer to blue than to red.
- As can be seen from the table below:
  - the average green wavelength of green – which is 532.5 nm,
  - differs from the average between blue and red, i.e., from the value $\dfrac{462.5 \text{ nm} + 687.5 \text{ nm}}{2} = 575$ nm, by about 8%.

8. A seemingly natural idea (cont-d)

- 8% may not sound big.
- But it *is* a big difference, if we take into account that the difference between blue and green is approximately 13%:

$$\frac{532.5 \text{ nm} - 462.5 \text{ nm}}{532.5 \text{ nm}} = \frac{70 \text{ nm}}{532.5 \text{ nm}} \approx 13\%.$$

color	wavelength (nm)	average wavelength	frequency (THz)	average frequency
blue	450-485	462.5	620-670	645
green	500-565	532.5	530-600	565
red	625-750	687.5	400-480	440

9. What we do in this talk

- In the previous slides, we simply considered the difference between the wavelengths.
- Intuitively, the difference may sound like a good first approximation.
- However, it is not directly related to what is the main purpose both of human vision and of our color computing: data processing.
- From the data processing viewpoint, we want to generate and process more information.
- So it makes more sense to consider this situation from the information viewpoint.
- This is what we do in this talk.
- Our conclusion is that from this viewpoint, the intermediate wavelength should indeed:
  - be closer to the shortest one (in our case, blue)
  - than to the longest one (in our case, red).

10. What we do in this talk (cont-d)

- The corresponding numerical model:
  - leads to the estimate of the intermediate wavelength
  - which is much closer to the observed one than the above-mentioned arithmetic average;
  - namely, it is 4% close instead of the original 8%.
- Can we get an even more accurate estimate?
- The problem is that the wavelengths corresponding to the three colors are only known approximately.
- Different sources cite somewhat different numbers, numbers differing by 5% and even sometimes more.
- From this viewpoint, 4% accuracy is the best we can achieve.

11. Biological meaning of our result

- From biological viewpoint, our result shows that:
  - there is a *fundamental* reason – and not any specific reason related to specific chemicals
  - why the intermediate color selected for human perception is closer to one of the other colors.
- In other words, our analysis shows that in this particular aspect, human perception is also close to optimal.
- This is in line with what we argued earlier about optimality of biological designs.

12. This result also (partially) explain the medical puzzle of color blindness

- Most people easily distinguish between the three basic colors.
- However, a reasonable proportion of people cannot distinguish between at least two of them.
- E.g., about 8% of the world's males have such a problem.
- This condition is – somewhat confusingly – known as *color blindness*.
- It is confusing since most patients are *not* totally blind to colors.
- They *can* distinguish some colors – although not all of them.
- Intuitively, the closer the two colors, the more probable it is that a somewhat faulty optical perception system would get confused.

13. Color blindness (cont-d)

- From this viewpoint:
  - since among the three basic colors, the closest pair is green and blue,
  - one would expect that the most frequent case of color blindness is the inability to distinguish between green and blue colors.
- Such a condition exists, but it is far from being the most frequent case.
- The most frequent case is red-green color blindness, when:
  - a patient cannot distinguish between green and red, while
  - he/she (it is usually a he) is perfectly capable of distinguishing blue from any of these two colors.
- In other words, we have a paradox.

14. Color blindness (cont-d)

- Namely:
  - the difference 687.5 nm − 532.5 nm = 165 nm between red and green wavelengths is more than twice larger than
  - the difference 532.5 nm − 462.5 nm = 70 nm between green and blue wavelengths.
- However, the red-green color blindness is much more frequent than the green-blue one.
- Our analysis shows that:
  - from the information-theoretic viewpoint,
  - green is almost as close to red as to blue.
- From this viewpoint, the difference between green and blue is not the smallest.
- So we no longer expect the corresponding color blindness to be the most frequent type.

15. Color blindness (cont-d)

- This is still a partial explanation.
- We still do not understand why:
  - the red-green color blindness is much more frequent than the green-blue one, while
  - from the information viewpoint, the corresponding pairs of colors are equally distant.

16. Natural criterion

- From the information viewpoint, a reasonable measure of an interval between two wavelengths is:
  - not the difference between these wavelengths,
  - but rather the overall amount of information (per unit time) that we can transmit by using wavelengths from this interval.
- So, it is reasonable to select the intermediate wavelength for which:
  - the amount of information corresponding to the interval $[\underline{\lambda}, \lambda]$ should be equal to
  - the amount of information corresponding to the interval $[\lambda, \overline{\lambda}]$.
- To describe this criterion in precise terms, let us estimate the amount of information corresponding to a generic interval $[\lambda^-, \lambda^+]$.

17. How much information can we transmit by using a single wavelength

- We want to come up with the desired estimate.
- Let us first analyze how much information we can transmit if we use a single wavelength λ.
- In principle, we can modulate signal of each cycle – be it amplitude modulation (AM) or frequency modulation (FM) or even both.
- From this viewpoint, the amount of information $I(\lambda)$ that we can transmit per second is proportional:
  - to the number of cycles per second,
  - i.e., to the frequency f of the signal:

$$I(\lambda) = A \cdot f \text{ for some constant } A.$$

- It is known that the frequency f is inverse proportional to the wavelength.

18. How much information can we transmit by using a single wavelength (cont-d)

- Namely, since the signal are traveling with the speed of light c, the duration of each cycle is equal to $T = \frac{\lambda}{c}$.
- Thus, the frequency is equal to the number of cycles in a second, i.e., to $f = \frac{1}{T} = \frac{c}{\lambda}$.
- Substituting this expression into the formula for I, we conclude that

$$I(\lambda) = \frac{C_1}{\lambda}, \text{ where we denoted } C_1 \stackrel{\text{def}}{=} A \cdot c.$$

19. In a given interval, how many distinguishable wavelengths can we use?

- How can we measure frequency?
- We let the signal go through some fixed number of cycles N, and then measure the overall time $t = N \cdot T$ that these cycles take.
- This means that we measure the value $t = N \cdot \dfrac{\lambda}{c}$.
- Let ε denoted the relative accuracy of measuring time.
- This means that we can distinguish between two wavelengths λ and $\lambda + \Delta\lambda$ is larger than or equal to $\varepsilon \cdot t$:

$$N \cdot \frac{\lambda + \Delta\lambda}{c} - N \cdot \frac{\lambda}{c} \geq \varepsilon \cdot N \cdot \frac{\lambda}{c}.$$

20. In a given interval, how many distinguishable wavelengths can we use (cont-d)

- Let us:
  - subtract the two fractions in the left-hand side,
  - divide both sides by N, and
  - multiply both sides by c.
- Then, we get an equivalent inequality $\Delta\lambda \geq \varepsilon \cdot \lambda$.
- So, the smallest distinguishable difference between two wavelengths is $\varepsilon \cdot \lambda$.
- Thus, on each wavelength interval of small width w, we have $\dfrac{w}{\varepsilon \cdot \lambda}$ distinguishable wavelengths.

21. We are now ready to estimate the overall amount of information

- Each of the distinguishable wavelengths is capable of transmitting the amount of information $I(\lambda)$ – as described by the above formula.
- Thus:
  - the overall amount of information that can be transmitted by wavelengths from the interval of small width w
  - can be obtained by multiplying number of distinguishable wavelengths by $I(\lambda)$.
- It is, thus, equal to $\dfrac{w}{\varepsilon \cdot \lambda} \cdot \dfrac{C_1}{\lambda} = \dfrac{C_1}{\varepsilon} \cdot \dfrac{1}{\lambda^2} \cdot w.$
- We want the overall amount of information I transmitted per second by all wavelengths from the interval $[\lambda^-, \lambda^+]$.
- This can obtained if we divide this interval into small subintervals and add up the corresponding expressions: $I = \sum \dfrac{C_1}{\varepsilon} \cdot \dfrac{1}{\lambda^2} \cdot w.$

22. We are now ready to estimate the overall amount of information (cont-d)

- This sum is an integral sum.
- The narrower subintervals we take, the close this sum to the integral.
- In the limit when $w \to 0$, we get $I = \int_{\lambda^-}^{\lambda^+} \frac{C_1}{\varepsilon} \cdot \frac{1}{\lambda^2}\, d\lambda$.
- This integral is easy to compute: it is equal to $I = \frac{C_1}{\varepsilon} \cdot \left(\frac{1}{\lambda^-} - \frac{1}{\lambda^+}\right)$.
- This formula becomes even simpler if we take into account that we have $\frac{1}{\lambda} = \frac{1}{c} \cdot f$.
- Let us denote:
  - by f^+ the frequency corresponding to the wavelength λ^-, and
  - by f^- the frequency corresponding to the wavelength λ^+.

23. We are now ready to estimate the overall amount of information (cont-d)

- Then, we get $I = \dfrac{C_1}{c \cdot \varepsilon} \cdot (f^+ - f^-)$.
- In other words:
  - the amount of information that can be transmitted by all the signals in the given wavelength interval
  - is proportional to the difference between the corresponding frequencies.

24. So, from the information viewpoint, what intermediate wavelength should we select

- Based on the formula, we conclude that:
  - for the intermediate wavelength,
  - the difference between its frequency f and each of two other frequencies $\underline{f}$ and $\overline{f}$ should be the same: $f - \underline{f} = \overline{f} - f$.
- Thus, as the intermediate frequency f, we should select the arithmetic average of two other frequencies:

$$f = \frac{\underline{f} + \overline{f}}{2}.$$

25. This indeed leads to a more accurate estimate

- For frequencies, the arithmetic average of the red and blue frequencies is equal to

$$\frac{440 \text{ THz} + 645 \text{ THz}}{2} = 542.5 \text{ THz}.$$

- The difference between this average and the actual green frequency of 565 THz is about 4%.
- It is twice smaller than when we naively averaged wavelengths.
- Let us transform these values into wavelengths.
- Then, we see that indeed, in terms of wavelengths:
  - the intermediate wavelength corresponding to the optimal frequency 542.5 THz
  - is closer to blue than to red.
- So, we have indeed explained why in terms of the wavelengths, green is closer to blue than to red.

26. Acknowledgments

This work was supported in part by:

- National Science Foundation grants 1623190, HRD-1834620, HRD-2034030, and EAR-2225395;
- AT&T Fellowship in Information Technology;
- a grant from the Hungarian National Research, Development and Innovation Office (NRDI), and
- by the Institute for Risk and Reliability, Leibniz University Hannover, Germany.