When Revolutions Happen: Algebraic Explanation

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1. When Revolutions Happen

- People usually believe that revolutions happen when life under the old regime becomes intolerable.
- However, a historical analysis shows that the usual understanding is wrong.
- Most revolutions happen *not* when the situation is at its worst.
- They usually happen when the situation has been improving for some time and then suddenly gets worse.
- Although, by the way, it never gets as bad as it was before the improvement started.

2. How Can We Explain This?

- Experiments show that in most situations, people act rationally:
  - the more their needs are satisfied, in general,
  - the happier they are.
- So why right before the revolution:
  - when the level of living is higher (often much higher) than in the recent past,
  - people are so much less happy that they start a revolution?
- How can we explain this unexpected (and somewhat counterintuitive) behavior?

3. Traditional Decision Theory: A Brief Reminder

- In traditional decision theory, people's preferences are described by numerical values called *utilities*.
- The actions of a person are determined:
  - not just by this person's current level of satisfaction – as described by the current utility value u_0,
  - but also by the expected future utility values u_1, u_2, etc.
- If we have m dollars, we can place it in a bank and get $(1+\alpha)^t \cdot m$ at time t, where α is the interest rate.
- Thus, \$1 at time t is equivalent to q^t dollars now, where $q \stackrel{\text{def}}{=} \dfrac{1}{1+\alpha}$.
- So, if we get m_0 now, m_1 in the next year, etc., this is equivalent to getting the following amount now:

$$m_0 + q \cdot m_1 + q^2 \cdot m_2 + \ldots$$

4. This General Approach Requires Extrapolation

- The future amounts are based on extrapolation:
  - we select a family of functions characterized by a few parameters $u_t = f(p_1, \ldots, p_n, t)$,
  - then we find the values $\widehat{p}_1, \ldots, \widehat{p}_n$ of the parameters that best fit the observed data u_0, u_{-1}, u_{-2}, etc.,
  - and then we use these values to predict future values as $f(\widehat{p}_1, \ldots, \widehat{p}_n, t)$.
- Let's use models that linearly depend on p_i:
  - then, matching parameters to data means easy-to-solve solving systems of linear equations,
  - while solving systems of nonlinear equations is, in general, NP-hard.
- Thus, we consider models $u_t = \sum\limits_{i=1}^n p_i \cdot f_i(t)$, where $f_i(t)$ are given functions, and p_i are parameters.

5. Which Basis Functions fi(t) Should We Choose?

- Most transitions are smooth; so, it's reasonable to require that all the functions $f_i(t)$ are smooth.
- Another reasonable requirement is related to the fact that the numerical value of time depends:
  - on the choice of a measuring unit – years or months,
  - and on the choice of a starting time.
- If we change a measuring unit by a new one which is a times smaller, then $t \to a \cdot t$.
- If we replace the original starting point with the new one, b units in the past $t \to t + b$.
- The general formulas for extrapolation should not depend on such an arbitrary things as:
  - selecting a unit of time or
  - selecting a starting point.
- It is therefore reasonable to assume that the approximating family $\left\{\sum\limits_{i=1}^n p_i \cdot f_i(t)\right\}$ will not change:

$$\left\{\sum_{i=1}^n p_i \cdot f_i(a \cdot t)\right\}_{p_1,\ldots,p_n} = \left\{\sum_{i=1}^n p_i \cdot f_i(t+b)\right\}_{p_1,\ldots,p_n} = \left\{\sum_{i=1}^n p_i \cdot f_i(t)\right\}_{p_1,\ldots,p_n}.$$

- It turns out that under these conditions, all the basic functions are polynomials.
- So, all their linear combinations are polynomials.
- Thus, it is reasonable to approximate the actual history by a polynomial.

6. Two Simple Situations

- We will compare two simple situations:
  - a situation in which the level of living is consistently bad $u_0 = u_{-1} = \ldots = u_{-k} = \ldots = c_1$ for small c_1;
  - a situation in which the level of living used to be much better, but now somewhat decreased:

$$u_{-1} = u_{-2} = \ldots = c_+ \text{ but } u_0 = c_- < c_+.$$

- In the first situation, of course, a reasonable extrapolation should lead to the exact same small value $u_0 = c$.
- Thus, the overall utility is equal to

$$u_0 + q \cdot u_1 + \ldots = c \cdot (1 + q + q^2 + \ldots) = \frac{c}{1-q}.$$

- But what to expect in the second situation?
- Let us start our analysis with the simplest possible linear extrapolation.

7. Linear Extrapolation

- In this case, we make our future predictions based only on two utility values: u_0 and u_{-1}.
- Since $u_0 < u_{-1}$, we get a linear decreasing function.
- Its values tend to $-\infty$ as the time t increases.
- So, when q is close to 1, the corresponding value

$$u_0 + q \cdot u_1 + \ldots \approx u_0 + u_1 + \ldots \text{ becomes negative.}$$

- This explains why in the second situation, the revolution is much more probable.

8. What About More Realistic Approximations?

- One may think that the above explanation is caused by our oversimplification of the extrapolation model.
- Of course, linear extrapolation is a very crude and oversimplified idea.
- What happens if we use higher degree polynomials for extrapolation?
- Let us assume that for extrapolation, we use polynomials of order d.
- The corresponding family of polynomials have d_1 parameters, so we can fit $d+1$ values $u_0, u_{-1}, \ldots, u_{-d}$.
- Let us find the polynomial $P(t)$ of degree d that fits these values: $P(0) = c_-$, $P(-1) = \ldots = P(-d) = c_+$.

Realistic Approximations (cont-d)

- For $Q(t) \stackrel{\text{def}}{=} P(t) - c_+$, we have $Q(-d) = \ldots = Q(-1) = 0$ and $Q(0) = c_- - c_+$.
- This polynomial of degree d has d roots $t = -1, \ldots, t = -d$, so $Q(t) = C \cdot (t+1) \cdot (t+2) \cdot \ldots \cdot (t+d)$, and

$$Q(t) = c_+ + (c_- - c_+) \cdot \frac{(t+1) \cdot (t+2) \cdot \ldots \cdot (t+d)}{1 \cdot 2 \cdot \ldots \cdot d}.$$

- Since $c_- < c_+$, this value is negative – and tends to $-\infty$ as the time t increases.
- In comparison with the linear extrapolation case, it tends to $-\infty$ even faster: as t^d.
- So, *the revolution phenomenon can be explained* no matter what degree of extrapolation we use.

9. Discussion

- We have explained the seemingly counterintuitive revolution phenomenon.
- Based on our analysis, we can make auxiliary conclusions (which also fit well with common sense).
- Revolutions only happen if people care about the future.
- If they don't, if $q \approx 0$, people are happy with their present-day level of living.
- The more into the past the people go in their analysis, the more probable it is that they will revolt.
- People who do not know their history are less prone to revolutions than people who do.

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11. Bibliography

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