Shall We Be Foxes or Hedgehogs: What Is the Best Balance for Research?

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1. Foxes and hedgehogs: a positive viewpoint

- In his famous essay, Isaiah Berlin, an American philosopher, divide all the thinkers into:
  - *hedgehogs*, who have one main idea (or a few main ideas) and apply it (them) to several problems, and
  - *foxes*, who have many different ideas.
- Some great thinkers were hedgehogs (Freud and Zadeh come to mind right away), some – like Aristotle – were foxes.
- At first glance, it looks like both types of thinkers could reach great results.
- But each of these two types has its limitations.

2. Foxes: a negative viewpoint

- At first glance, what can be wrong with having many interesting ideas, with always learning many interesting ideas?
- Well, the problem is that you may spread yourself too thin.
- For example, in mathematical logic, Georg Kreisel was one of the most productive authors, publishing many papers with interesting ideas.
- This did not bother hedgehogs.
- However, several foxes – eagerly interested in learning new ideas – complained that:
  - they have no time to do their own research:
  - they have to read all new papers by Kreisel.

3. Hedgehogs: a negative viewpoint

- Lotfi Zadeh was clearly a hedgehogs.
- However, he liked to emphasize what can go wrong with this approach, by reminding us of the saying that:
  - if all you have is a hammer,
  - then everything starts looking like a nail.
- We have seen many examples of this:
  - in politics, when an originally successful idea gets used everywhere;
  - in popular medicine, where successful medicines like antibiotics gets too overused, etc.
- In Russia, where several of us are from, we had a silly joke showing this problem.

4. Hedgehogs: a negative viewpoint (cont-d)

- A young man wants to become a writer, so he is taking an entrance exam to the writer's program.
- – What can you say about Tolstoy's War and Peace?
- – Never read it.
- – ??? Did not you say that you want to become a writer?
- – Yes, but I want to be a writer, not a reader.
- In science, some hedgehogs become such writers-not-readers:
  - they may have a had a great idea,
  - but later on, their reluctance to adopt new ideas makes them not very productive.

5. Hedgehogs: a negative viewpoint (cont-d)

- This even happened to great Einstein.
- He started as a fox – e.g., his Nobel prize was for photo-effect, not for relativity.
- However, who spent several not-very-productive last decades on a single not-very-successful idea of a unified field theory.

6. There should be a balance, but what is this balance?

- Both extremes can be counterproductive.
- So, there should be a balance between these two extremes, a balance that leads to the maximal possible productivity.
- In this talk:
  - we provide a simple model of the situation, and
  - we use this model to provide recommendations on the best balance.

7. We need to generate new ideas

- The whole idea of research is to solve problems that no one was able to solve before.
- This means that the existing ideas are not enough to solve the corresponding problem.
- We need to have a new idea, or at least a new twist on an existing idea.

8. Generating ideas: notations

- Let us assume that a researcher spend time t_I on developing a new idea (or a new twist on a new idea); then:
  - if during a certain period of time T_0, the researcher comes up with I ideas,
  - then overall, during this period, this researcher spends time $T_I = t_i \cdot I$ on coming up with new ideas.
- For a hedgehog, $I \approx 1$.
- For a fox, the number of new ideas I is much larger than 1: $I \gg 1$.

9. Understanding problems: notations

- To be able to solve a problem, it is important to spend some time understanding this problem.
- This is not easy – especially if this problem is from an area which is different from the researcher's main area of expertise.
- Let us denote the average time needed to understand a problem by t_P.
- Let us denote the number of different problems the researcher learns during the period T_0 by P.
- Then overall, during this period, the researcher spends time $T_P = t_P \cdot P$ on learning new problems.

10. We need to apply these ideas

- The whole purpose of coming up with new ideas is to solve problems.
- And the whole purpose of learning a problem is to try to solve it.
- If one idea is not working on a problem, a reasonable approach is to apply a different idea.
- Some problems are solved, most are not – unless we are dealing with a genius who solves all the problems, and such geniuses are rare.
- In general, a researcher applies all his/her ideas to all the problems that he/she tries to solve.
- Indeed, what is the purpose of learning a new problem if you do not try to solve it by using all ideas you have?
- Let t_0 denote the time that it takes, on average, to try one idea on one problem.
- Then, to try each of I ideas on each of P problems, we need time

$$t_0 \cdot I \cdot P.$$

11. Resulting constraint

- The overall time that a researcher spends cannot exceed T_0.
- This is spent on:
  - inventing ideas,
  - learning the problems, and
  - trying ideas on problems.
- Thus, we have the following constraint:

$$t_I \cdot I + t_P \cdot P + t_0 \cdot I \cdot P \le T_0.$$

12. What do we want?

- The main objective of research is to solve problems.
- The more problems we solve altogether, the more successful we are in our research efforts.
- From this viewpoint, we should therefore aim for maximizing the number of solved problems.

13. How many problems can we solve this way?

- A priori, we do not know which idea will work on which problem.
- So, it is natural to assume that:
  - for each pairs of an idea and a problem,
  - there is the same probability that this particular idea will solve this particular problem.
- This assumption is known as Laplace Indeterminacy Principle.
- Let p_0 denote this joint probability.
- This probability means that out of all $I \cdot P$ pairs, the proportion of those that lead to solution is equal to p_0.
- Thus, the overall number of problems solved by a researcher is equal to $p_0 \cdot I \cdot P$.
- So, we arrive at the following optimization problem.

14. Resulting optimization problem

- Let us assume that we are planning for time period T_0.
- For a given researcher, we know:
  - the average time t_I that it takes this researcher to come up with a new idea or a new twist on an idea;
  - the average time t_P that it takes this researcher to understand a new problem;
  - the average time t_0 that it takes this researcher to apply an idea to a problem; and
  - the probability p_0 that a randomly selected idea will solve a randomly selected problem.
- We want to find the number of ideas I and the number of problems P that maximize the expected number of solved problems.
- Let us now solve this problem.

15. First simplification

- Suppose that in the time constraint, we have a strict inequality.
- This would mean that we can increase either I or P (or both) without violating the constraint.
- Thus, we would increase the value of the objective function.
- Thus, the maximum of the objective function is attained when in the time constraint, we have equality, i.e., when

$$t_I \cdot I + t_P \cdot P + t_0 \cdot I \cdot P = T_0.$$

- So, we have a problem of optimizing the objective function $p_0 \cdot I \cdot P$ under this constraint.

16. Second simplification

- In terms of T_I and T_P, we have

$$I = \frac{T_I}{t_I}, \quad P = \frac{T_P}{t_P}, \text{ and thus, } t_0 \cdot I \cdot P = c \cdot T_I \cdot T_P.$$

- Here, we denoted

$$c \stackrel{\text{def}}{=} \frac{t_0}{t_I \cdot t_P}.$$

- In these terms, the time constraint takes the form

$$T_I + T_P + c \cdot T_I \cdot T_P = T_0.$$

- The objective function takes the form

$$p_0 \cdot I \cdot P = c_0 \cdot T_I \cdot T_P, \text{ where } c_0 \stackrel{\text{def}}{=} \frac{p_0}{t_I \cdot t_P}.$$

- Let us maximize this objective function under this constraint.

17. Let us use Lagrange multiplier method

- For some λ, the original constrained optimization problem is equivalent to the unconstrained problem of optimizing the expression

$$c_0 \cdot T_I \cdot T_P + \lambda \cdot (T_I + T_P + c \cdot T_I \cdot T_P - T_0).$$

- For an unconstrained optimization problem, maximum is attained when all the partial derivatives are equal to 0.
- Differentiation with respect to T_I and equating the derivative to 0, we conclude that $c_0 \cdot T_P + \lambda + \lambda \cdot c \cdot T_P = 0$.
- Hence $T_P \cdot (c_0 + \lambda \cdot c) = -\lambda$, and $T_P = -\dfrac{\lambda}{c_0 + \lambda \cdot c}$.
- Similarly, differentiation with respect to T_P and equating the derivative to 0, we conclude that $c_0 \cdot T_I + \lambda + \lambda \cdot c \cdot T_I = 0$.
- Hence $T_I \cdot (c_0 + \lambda \cdot c) = -\lambda$ and $T_I = -\dfrac{\lambda}{c_0 + \lambda \cdot c}$.

18. First conclusion

- By comparing the above expressions, we conclude that we have

$$T_I = T_P.$$

- So, *the time spent on inventing new ideas should be equal to the time spent on learning new problems.*

19. So fox or hedgehog?

- From the above solution, we conclude that $I = \dfrac{t_P}{t_I} \cdot P$.
- So:
  - for researchers for whom $t_P \ll t_I$,
  - i.e., for whom it is much easier to understand a new problem than to come up with a new idea,
  - it is better to generate fewer ideas but apply them to many problems,
  - in other words, to be a hedgehog.

20. So fox or hedgehog (cont-d)

- On the other hand:
  - for researchers for whom $t_I \ll t_P$,
  - i.e., for whom it is much easier to come up with a new idea that to understand a new problem,
  - it is better to generate many ideas but apply them to fewer problems,
  - in other words, to be a fox.
- For the cases when the times t_I and t_P are of the same order, the above provides the desired optimal balance.

21. So what are the optimal values of P and I?

- In the optimal case, we have $T_I = T_P$.
- So, the time constraint takes the form $2T_I + c \cdot T_I^2 = T_0$.
- By solving this quadratic equation, we get

$$T_I = T_P = \frac{\sqrt{1 + c \cdot T_0} - 1}{c}, \text{ thus}$$

$$I = \frac{T_I}{t_I} = \frac{t_P}{t_0} \cdot \left(\sqrt{1 + \frac{t_0}{t_I \cdot t_P} \cdot T_0} - 1 \right) \text{ and}$$

$$P = \frac{T_P}{t_P} = \frac{t_I}{t_0} \cdot \left(\sqrt{1 + \frac{t_0}{t_I \cdot t_P} \cdot T_0} - 1 \right).$$

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