Kuznets Curve: A Simple Dynamical System-Based Explanation

Thongchai Dumrongpokaphan[1] and Vladik Kreinovich[2]

[1]Department of Mathematics, Chiang Mai University, Thailand
tcd43@hotmail.com
[2]University of Texas at El Paso, El Paso, Texas 79968, USA
vladik@utep.edu

1. What Is Kuznets Curve

- In the 1950s, Simon Kuznets, an American economist of Russian origin, showed that:
  - as the country's Gross Domestic Product (GDP) increases,
  - inequality first increases and then decreases again.
- The resulting dependence on inequality on GDP looks like an inverted letter U.
- It is thus called an *inverted U-shaped* dependence or the *Kunzets curve*.
- For this work, Professor Kuznets was awarded a Nobel Prize in Economics in 1971.

2. Kuznets Curve: A Controversy

- The Kuznets curve is a purely empirical observation.
- Free-market champions use it as an argument that the governments should not interfere with economy:
  - inequality will decrease by itself,
  - as soon as the economy improves further.
- So, they recommend to minimize government interventions.
- As Ronald Reagan used to say, "The rising tide lifts all the boats".

3. Kuznets Curve: A Controversy (cont-d)

- Other economists notice that:
  - in all developed countries,
  - there was a lot of government intervention aimed at decreasing inequality.
- So, they recommend government regulations.
- Additionally, some researchers doubt that the Kunzets curve is indeed a universal phenomenon.
- We show that the Kuznets curve phenomenon naturally follows from the general system-based analysis.

4. Analysis of the Problem

- We start in a situation when the overall economic output is small and therefore, most everyone is poor.
- In such situations, while there may be a small minority of relatively rich people, most people are poor.
- In this sense, there is not much inequality.
- As the economy grows, people's incomes grow.
- Each person's income grows until it reaches some level m_i.
- This level expressing the capability of this person to earn money in the corresponding economy.
- People are different, so they have somewhat different rates v_i at which they move towards this larger income.
- For simplicity, we can assume that for each person, the rate does not change with time.

5. Analysis of the Problem (cont-d)

- So, at each moment of time t, the income $x_i(t)$ of the i-th person is:
  - equal to $x_i(t) = v_i \cdot t$ when $t \le \dfrac{m_i}{v_i}$, and
  - equal to $x_i(t) = m_i$ for $t \ge \dfrac{m_i}{v_i}$.
- The values m_i are centered around the mean $\overline{m}$, with random deviations $\Delta m_i \stackrel{\text{def}}{=} m_i - \overline{m}$.
- Similarly, the rates v_i center around the mean $\overline{v}$, with random deviations $\Delta v_i \stackrel{\text{def}}{=} v_i - \overline{v}$.
- There is no reason to believe that there is a correlation between m_i and v_i.
- So, we will assume these variables to be independent.

6. How Can We Describe Inequality

- Perfect equality means that everyone's income is the same.
- This is equivalent to saying that the standard deviation of income is 0.
- In general,
  - if the standard deviation is equal to 10% of the average income,
  - then we have less inequality that when the standard deviation is 20% of the average income.
- Thus, a natural measure of inequality is the ratio between the income's standard deviation and its mean.
- Now, we are ready to analyze how inequality changes when the economy improves.

7. Stages

- Kuznets curve considers three stages:
  - the starting stage, when the inequality level is relatively low,
  - the intermediate stage, when the level of inequality increases, and
  - the final stage, when the level of inequality decreases.
- We have already discussed that in the beginning, there is practically no inequality.
- So, to complete our analysis, we need to consider two other stages: the intermediate stage and the final stage.

8. What Happens on the Final Stage

- On the final stage, everyone reaches their potential m_i; thus:
  - the average income is equal to the average $\overline{m}$ of the values m_i, and
  - the standard deviation is equal to the standard deviation σ_m of the differences Δm_i.
- So, on the final stage, the inequality level is equal to the ratio $\dfrac{\sigma_m}{\overline{m}}$.

9. What Happens on the Intermediate Stage

- In the beginning of the intermediate stage, the income of each person is equal to

$$x_i(t) = \frac{m_i}{v_i} \cdot t = \frac{\overline{m} + \Delta m_i}{\overline{v} + \Delta v_i} \cdot t = \frac{\overline{m}}{\overline{v}} \cdot t \cdot \frac{1 + \dfrac{\Delta m_i}{\overline{m}}}{1 + \dfrac{\Delta v_i}{\overline{v}}}.$$

- Differences between different people are, in most cases, not so large, so $|\Delta m_i| \ll \overline{m}$ and $\dfrac{\Delta m_i}{\overline{m}} \ll 1$.
- Similarly, $|\Delta v_i| \ll \overline{v}$ and thus, $\dfrac{\Delta v_i}{\overline{v}} \ll 1$.
- Thus, we can expand the expression for $x_i(t)$ in $\dfrac{\Delta m_i}{\overline{m}}$ and $\dfrac{\Delta v_i}{\overline{v}}$ and keep only linear terms:

$$x_i(t) = \frac{\overline{m}}{\overline{v}} \cdot t \cdot \left(1 + \frac{\Delta m_i}{\overline{m}} - \frac{\Delta v_i}{\overline{v}}\right).$$

10. Intermediate Stage (cont-d)

- *Reminder:* $x_i(t) = \dfrac{\overline{m}}{\overline{v}} \cdot t \cdot \left(1 + \dfrac{\Delta m_i}{\overline{m}} - \dfrac{\Delta v_i}{\overline{v}}\right)$.
- The mean value of Δm_i and Δv_i is 0, so the mean income is equal to $\overline{x}(t) = \dfrac{\overline{m}}{\overline{m}} \cdot t$.
- The standard deviation of Δm_i is equal to σ_m, so the standard deviation of the ratio $\dfrac{\Delta m_i}{\overline{m}}$ is equal to $\dfrac{\sigma_m}{\overline{m}}$.
- Similarly, the standard deviation of the ratio $\dfrac{\Delta v_i}{\overline{v}}$ is equal to $\dfrac{\sigma_v}{\overline{v}}$.
- The quantities Δm_i and Δv_i are assumed to be independent; thus: $\dfrac{\sigma_x(t)}{\overline{x}(t)} = \sqrt{\dfrac{\sigma_m^2}{(\overline{m})^2} + \dfrac{\sigma_v^2}{(\overline{v})^2}}$.

11. Comparison of Two Stages

- On the final stage, we have $\dfrac{\sigma_x(t)}{\overline{x}(t)} = \dfrac{\sigma_m}{\overline{m}}$.
- On the intermediate stage, we have

$$\frac{\sigma_x(t)}{\overline{x}(t)} = \sqrt{\frac{\sigma_m^2}{(\overline{m})^2} + \frac{\sigma_v^2}{(\overline{v})^2}}.$$

- Clearly, $\sqrt{\dfrac{\sigma_m^2}{(\overline{m})^2} + \dfrac{\sigma_v^2}{(\overline{v})^2}} > \dfrac{\sigma_m}{\overline{m}}$.
- Thus, the inequality is smaller on the final stage.
- This is exactly the Kuznets curve phenomenon.
- So, we have indeed arrived at a simple justification of the Kuznets curve phenomenon.

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