Prediction in Econometrics: Towards Mathematical Justification of Simple (and Successful) Heuristics

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1. Prediction is Important

- Prediction (forecasting) is of upmost importance in economics and finance.
- If we can accurately predict the future prices, then we can get the largest return on investment.
- Vice versa:
  - if we make decisions based on the wrong predictions,
  - then our financial investments collapse,
  - and the manufacturing plants that we built are non-profitable and thus idle.
- Many successful (semi-)heuristic methods have been proposed to predict economic and financial processes.

2. Need to Justify Heuristic Strategies

- The success of prediction heuristics leads to a conjecture that these heuristics have a theor. justification.
- In general, when we have a theoretical justification, it helps:
  - we can use the corresponding theory to fine-tune the method, and
  - we can get a clearer understanding of when the method is efficient and when it is not efficient.
- In this paper, we justify two heuristics:
  - of an intuitive exponential smoothing procedure, that predicts slowly changing processes, and
  - of a seemingly counter-intuitive idea of an increase in volatility as a predictor of trend reversal.

3. First Result: Prediction of Slowly Changing Processes

- *Problem:* based on the past observations $x_1, \ldots, x_T$ (x_1 most recent), predict the future value x_0.
- *In other words:* we need a predictor function $x_0 \approx F(x_1, \ldots, x_T)$.
- *Continuity:* if $x_i \approx x'_i$, then $F(x_1, \ldots, x_T) \approx F(x'_1, \ldots, x'_T)$.
- *Motivation:* we predict based on measurement results, and they are never absolutely accurate.
- *Additivity:* $F(x_1^{(1)} + x_1^{(2)}, \ldots) = F(x_1^{(1)}, \ldots) + F(x_1^{(2)}, \ldots)$.
- *Motivation:* we can predict stocks $x_0^{(1)}$ and bonds $x_0^{(2)}$, or we can predict value of the whole portfolio $x_0^{(1)} + x_0^{(2)}$.
- *Conclusion:* we must consider linear predictors $F(x_1, \ldots, x_T) = \sum\limits_{t=1}^{T} f_t \cdot x_t$.

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4. From Finite to Infinite Time

- The actual number of observed values is always finite.
- However, in many cases, we have very long time series (e.g., daily for many years).
- In real life, the influence of remote events is small.
- It is thus reasonable to assume that we have an infinite number of records: $x_0 = \sum_{t=1}^{\infty} f_t \cdot x_t$.
- In practice, we only know values $x_1, \ldots, x_T$.
- Thus, we use an approximate formula

$$x_0 \approx \sum_{t=1}^{T} f_t \cdot x_t.$$

5. Case of a Constant Signal

- In some cases, the observed signal x_t does not change at all: $x_t = c$.
- In this case, it is reasonable to predict the same value $x_0 = c$.
- In other words, if $x_1 = x_2 = \ldots = c$, then

$$x_0 = \sum_{t=1}^{\infty} f_t \cdot x_t = c.$$

- In precise terms, this means that $\sum\limits_{t=1}^{\infty} f_t \cdot c = c$.
- In particular, for $c = 1$, we get $\sum\limits_{t=1}^{\infty} f_t = 1$.

6. Exponential Smoothing: a Brief Reminder

- *General formula:* $x_0 = \sum\limits_{t=1}^{\infty} f_t \cdot x_t$.
- *Question:* which predictor is the best?
- *Empirical fact:* exponential smoothing is one of the best in econometrics: $f_t = \alpha \cdot (1-\alpha)^{t-1}$.
- *It is widely used:* described in textbooks, used in serious econometric studies.
- *Why exponential smoothing?* there exist many explanations for the usefulness of exponential smoothing.
- *Remaining problem:* these explanations are based on complex, not very intuitively clear statistical models.
- *What we do:* we provide a new (and rather simple) theoretical explanation of exponential smoothing.

7. Definitions

- By a *time series* x, we mean an infinite sequence of real numbers $x_1, \ldots, x_n, \ldots$
- By a *predictor function* f, we mean an infinite sequence of real numbers $f_1, \ldots, f_n, \ldots$ for which

$$\sum_{t=1}^{\infty} f_t = 1.$$

- By the *prediction* $X_0(f, x)$ made by the predictor function f_t for the time series x_t, we mean the value

$$\sum_{t=1}^{\infty} f_t \cdot x_t.$$

- By a *noise pattern* p, we mean a finite sequence of real numbers $p_1, \ldots, p_k$.

8. Definitions (cont-d)

- Let c be a real number, and let m be a natural number.
- By $x(p, c, m)$, we mean a time series for which $x_{m+i} = p_i$ for $i = 1, \ldots, k$, and $x_t = c$ for all other t.
- We say that this time series $x(p, c, m)$ corresponds to
  - a *a constant signal plus*
  - *a noise pattern p before moment m.*
- We say that for a predictor function f_t, *the effect of noise always decreases with time* if:
  - for every noise pattern p, for every real number c and
  - for every two natural numbers $m > m'$,

  we have

$$|X_0(f, x(p, c, m)) - c| \le |X_0(f, x(p, c, m')) - c|.$$

9. Our First Result: A Simple Justification of Exponential Smoothing

- *Result:*
  - For every $\alpha \in (0, 2)$, for $f_t = \alpha \cdot (1 - \alpha)^{t-1}$, the effect of noise always decreases with time.
  - If for a function f_t, the effect of noise always decreases with time, then there exists $\alpha \in (0, 2)$ s.t.:

$$f_t = \alpha \cdot (1 - \alpha)^{t-1}.$$

- *Discussion:*
  - Exponential smoothing is the only predictor for which the effect of noise always decreases with time.
  - Thus, the need to satisfy this natural property explains the efficiency of exponential smoothing.

10. Price Transmission: Reminder

- The price of a manufacturing product is determined by the price of the components and the price of the labor.
- If one of the component prices changes, this change affects the product's price.
- This change is called *price transmission*.
- Example:
  - when the oil price changes, the gasoline prices change as well;
  - when the gasoline prices change, the transportation prices change as well;
  - when the transportation prices change, the price of transported goods changes.

11. Asymmetric Price Transmission

- When the component (input) price increases, the final product (output) price starts increasing right away.
- On the other hand, when the input price starts decreasing back, the output price decreases much slower.
- As a result:
  - when the input price falls to the original lower level,
  - the output price remains much higher than the original one.
- This phenomenon seems to contradict to the usual economic assumption:
  - that markets are efficient, and
  - that the price of each product is determined by the equilibrium of supply and demand.

12. Asymmetric Price Transmission: A Problem

- *Good news:* there exist explanations of this phenomenon.
- *Problem:* these explanations are based on complex models and are far from intuitive clarity.
- *What we do:* provide a simple explanation for asymmetric price transfer.
- *Our explanation:* the output price is determined not by the *current* input price, but by the *predicted* price.
- *Example:* suppose that oil was \$20/barrel, then shoots to \$100, then goes down to \$20 and stays at \$20.
- *Predicted price:* for simplicity, $x_0 = (x_1 + x_2)/2$.
- First, it is $(20 + 20)/2 = 20$, then $(20 + 100)/2 = 60$, then $(100+20)/2 = 60$, and only then $(20+20)/2 = 20$.
- *Result:* a one year delay in price decrease.

13. Additional Intuitive Arguments in Favor of Our Explanation

- *Situation:*
  - the price of the component remains stable and
  - then experiences a sudden decrease.
- *Our prediction:* a sudden decrease in the customer price of a final product as well.
- *We indeed observe* such a phenomenon on the example of consumer electronics:
  - when the computer chips become cheaper,
  - many electronic products become cheaper as well.
- *In real life*, due to inflation, cases when consumer prices go down are rarer.

14. Second Example: Predicting Trend Reversal

- So far, we described the problem of predicting the new value when we are within a certain trend.
- Another important problem is predicting when a trend will reverse (e.g., when recession will end).
- It is a known empirical fact that volatility tends to increase before trend reversals.
- Thus, such volatility increases are a known predictor of trend reversals.
- This empirical fact is somewhat counter-intuitive:
  - when the trend changes from decrease to increase, the corresponding quantity reaches its minimum;
  - at the minimum, derivative is 0 – i.e., the local change is the smallest;
  - in economics, vice versa, the local change is the largest when trend reverses.

15. Towards an Explanation

- As an example of a time series, let us consider a stock price.
- With respect to the given stock, some traders are optimistic, some are pessimistic.
- An *optimistic* trader:
  - believes that the stock will rise,
  - so he/she is willing to pay a little extra for this stock,
  - in the expectation of larger gains in the future.
- A *pessimistic trader:*
  - believes that the stock will go down,
  - so he/she is willing to sell this stock even for a lower price than most.

16. Towards an Explanation (cont-d)

- The overall price of the stock can be computed as an average over all the transactions.
- Let x be the last recorded price for the stop.
- Let δ be the average value of the small increase/decrease in stock in transactions by optimists and pessimists.
- We are interested in the average behavior of all the traders in the market.
- In such an average behavior, individual differences tend to average out.
- Thus, it seems safe to simply assume that:
  - each optimist performs transactions with this stock at the price $x + \delta$, while
  - each pessimist performs transactions at the price $x - \delta$.

17. Towards an Explanation (cont-d)

- Let p be the proportion of optimists, i.e., the probability that a randomly selected trader is an optimist.
- To further simplify our description, we will also assume that all the traders are independent from each other.
- Let n denote the total number of traders. Thus, we arrive at the following model.
- We start with the price x.
- At the next moment of time, we have a price

$$x' = \frac{x_1 + \ldots + x_n}{n}, \text{ where:}$$

  - $x_i = x + \eta_i \cdot \delta$, and
  - $\eta_i = \pm 1$, with $\text{Prob}(\eta_i = 1) = p$.

18. Analysis of the Resulting Model

- *Reminder:* $x' = \dfrac{x_1 + \ldots + x_n}{n}$, where $x_i = x + \eta_i \cdot \delta$ and $\text{Prob}(\eta_i = 1) = p$.
- *Here:* $E[x'] = E[x_i] = x + (p \cdot 1 + (-1) \cdot (1-p))$, so the mean price is $E[x'] = x + (2p-1) \cdot \delta$.
- *Conclusion:* the price increases when $p > 1/2$ and decreases when $p < 1/2$.
- *Conclusion:* the trend reverses when $p = 1/2$.
- *Natural measure of volatility:* standard deviation σ.
- *Result of computations:* $\sigma = 2 \cdot \delta \cdot \dfrac{1}{\sqrt{n}} \cdot \sqrt{p \cdot (1-p)}$.
- *Interesting:* the value σ is the smallest when $p = 0$ and $p = 1$ and attains its largest value when $p = 1/2$.
- *Conclusion:* volatility is indeed the largest when the trend reverses – exactly as empirically observed.

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20. Appendix: Main Idea Behind the Proof of the Main Result

- *Main requirement:* the effect of noise pattern decreases with time.
- *Example:* two-value pattern (p_1, p_2) with $p_2 = 1$.
- *Requirement:* $|f_{m+1} \cdot p_1 + f_{m+2}| \le |f_m \cdot p_1 + f_{m+1}|$.
- *Hence:* for $p_1 = -\dfrac{f_{m+1}}{f_m}$, we have $f_m \cdot p_1 + f_{m+1} = 0$.
- *Conclusion:* we must have $f_{m+1} \cdot p_1 + f_{m+2} = 0$.
- *Conclusion:* $p_1 = -\dfrac{f_{m+1}}{f_m} = -\dfrac{f_{m+2}}{f_{m+1}}$.
- *By induction:* we get $\dfrac{f_2}{f_1} = \dfrac{f_3}{f_2} = \ldots = \dfrac{f_{m+1}}{f_m} = \ldots$
- *Thus:* $f_t = f_1 \cdot (1-\alpha)^{t-1}$, where $\alpha = 1 + p_1$.
- *Here:* $\sum\limits_{t=1}^{\infty} f_t = 1$ leads to $f_1 = \alpha$, so $f_t = \alpha \cdot (1-\alpha)^{t-1}$.