Why Unexpectedly Positive Experiences Make Decision Makers More Optimistic: An Explanation

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1. Formulation of the Problem

- Experiments show that unexpectedly positive experiences make decision makers more optimistic.
- This was first observed on rats: rats like being tickled, and tickled rats became more optimistic.
- Several later papers showed that the same phenomenon holds for other decision making situations as well.
- Similarly, decision makers who had an unexpectedly negative experiences became more pessimistic.
- There seems to be no convincing explanation for this experimental fact.
- We show that this phenomenon can be explained in the traditional utility-based decision theory.

2. What Does Optimism Mean?

- Traditional decision theory assumes that we know the probabilities of all possible consequences of each action.
- Then, a rational decision maker maximizes the expected value $u(a)$ of a special function called *utility*.
- In this case, there is no such thing as optimism or pessimism: we just select the best alternative a.
- In practice, we often have only *partial* information about these probabilities.
- In such situations, there are several possible probability distributions consistent with our knowledge.
- For different distributions, we have, in general, different values of the expected utility.
- As a result, for each alternative a, we have an *interval* $[\underline{u}(a), \overline{u}(a)]$ of possible values of $u(a)$.

3. What Does Optimism Mean (cont-d)

- In this case, we should select an alternative a that maximizes $u(a) = \alpha \cdot \overline{u}(a) + (1 - \alpha) \cdot \underline{u}(a)$.
- This idea was proposed by the Nobelist Leo Hurwicz.
- The selection of α, depends on the person.
- The value $\alpha = 1$ means that the decision maker only takes into account the best possible consequences.
- In other words, the values $\alpha = 1$ corresponds to complete optimism.
- Similarly, the value $\alpha = 0$ corresponds to complete pessimism.
- The larger α, the close this decision maker to complete optimism.
- The *optimism-pessimism index* α is a numerical measure of the decision maker's optimism.

4. What Does Optimism Mean (cont-d)

- The *optimism-pessimism index* α is a numerical measure of the decision maker's optimism.
- Thus, the phenomenon to-be-explained takes the following precise meaning:
  - if a decision maker has unexpectedly positive experiences, then this decision maker's α increases;
  - if a decision maker has unexpectedly negative experiences, then this decision maker's α decreases.

5. α Can Be Interpreted as the Subjective Probability of Positive Outcome

- The decision maker selects an alternative a that maximizes $\alpha \cdot \overline{u}(a) + (1 - \alpha) \cdot \underline{u}(a)$.
- Here, $\overline{u}(a)$ corresponds to the positive outcome, and $\underline{u}(a)$ corresponds to negative outcome.
- For simplicity, let us consider the situation when we have only two possible outcomes:
  - the positive outcome, with utility $\overline{u}(a)$, and
  - the negative outcome, with utility $\underline{u}(a)$.
- A traditional approach to decision making assumes that we know the probabilities of different outcomes.
- In this case of uncertainty, we do not know the actual (objective) probabilities.

6. α Can Be Interpreted as the Subjective Probability of Positive Outcome (cont-d)

- In the case of uncertainty, we do not know the actual (objective) probabilities.
- However, we can always come up with estimated (subjective) ones.
- Let us denote the subjective probability of the positive outcome by p_+.
- Then, the subjective probability of the negative outcome is equal to $1 - p_+$.
- The expected utility is equal to $p_+ \cdot \overline{u}(a) + (1 - p_+) \cdot \underline{u}(a)$.
- This is exactly what we optimize when we use Hurwicz's approach, with $\alpha = p_+$.
- Thus, the value α can be interpreted as the subjective probability of the positive outcome.

7. A New Reformulation of Our Problem

- Unexpectedly positive experiences increase the subjective probability of a positive outcome.
- Unexpectedly negative experiences decrease the subjective probability of a positive outcome.
- To explain this phenomenon, let us recall where subjective probabilities come from.
- If we observe an event in n out of N cases, our estimate is n/N.
- Example: if a coin fell heads 6 times out of 10, we estimate the probability of it falling heads as $6/10$.
- Let us show that this leads to the desired explanation.

8. Resulting Explanation

- Suppose that a decision maker had n positive experiences in the past N situations.
- Then, the decision maker's subjective probability of a positive outcome is $p_+ = n/N$.
- Unexpectedly positive experiences means that:
  - we have a series of new experiments,
  - in which the fraction of positive outcomes was higher than the expected frequency p_+.
- In other words, unexpectedly positive experiences means that $n'/N' > p$, where:
  - N' is the overall number of new experiences, and
  - n' is the number of those new experiences in which the outcome turned out to be positive.

9. Resulting Explanation (cont-d)

- The new subjective probability p'_+ is equal to the new ratio $p'_+ = \dfrac{n + n'}{N + N'}$.
- Here, by definition of p_+, we have $n = p_+ \cdot N$.
- Due to unexpected positiveness of new experiences, we have $n' > p_+ \cdot N'$.
- By adding this inequality and the previous equality, we conclude that $n + n' > p_+ \cdot (N + N')$, i.e., that

$$p'_+ = \frac{n + n'}{N + N'} > p_+.$$

- In other words, unexpectedly positive experiences increase the subjective probability of a positive outcome.

10. Resulting Explanation (final)

- The subjective probability of the positive outcome is exactly the optimism-pessimism coefficient α.
- Thus, $p'_+ > p_+$ means that $\alpha' > \alpha$.
- So, unexpectedly positive experiences make the decision maker more optimistic.
- Similarly, if we had unexpectedly negative experiences, i.e., $n' < p_+ \cdot N'$, then $p'_+ = \dfrac{n + n'}{N + N'} < p_+$ and $\alpha' < \alpha$.
- So, we conclude that unexpectedly negative experiences make the decision maker less optimistic.
- This is also exactly what we observe.
- So, we have the desired explanation.

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