

# Working on One Part at a Time is the Best Strategy for Software Production: A Proof

Francisco Zapata<sup>1</sup>, Maliheh Zargaran<sup>2</sup>, and  
Vladik Kreinovich<sup>2</sup>

<sup>1</sup>Department of Industrial, Manufacturing, and  
Systems Engineering

<sup>2</sup>Department of Computer Science  
University of Texas at El Paso  
El Paso, TX 79968, USA

fazg74@gmail.com, mzargaran@miners.utep.edu  
vladik@utep.edu

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# 1. It Is Possible to Start Earning Money Before the Whole Software Package Is Released

- When a company designs a software package, usually,
  - it does not have to wait until the whole package is fully functional to profit from sales:
  - the company can often start earning money once some useful features are implemented.
- As an example, let us consider a company that designs a package for all kinds of numerical computations.
- This package includes solving systems of equations, optimization, etc.
- The company does not have to wait until all the parts of the software are ready.
- The company can first release – and start selling – the parts that solve systems of equations.

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## 2. Earning Money Before the Package Is Released (cont-d)

- Thus, the company can start earning money before the whole package is ready for use.
- This possibility is critical.
- Indeed, for large software packages, full design can take years; so:
  - the possibility to recoup at least some of the original investment earlier
  - makes such long-term projects more acceptable to managers and shareholders.
- Thus, it makes these projects more probable to be approved.

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### 3. In View of This Possibility, What Is The Optimal Release Schedule for Different Parts?

- How can we best take into account the possibility to earn money
  - before the package is fully ready,
  - when only some parts of it are ready?
- What is the optimal schedule for releasing different parts?
- In what order should we work on them?

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## 4. Let Us Formulate This Problem in Precise Terms

- The software projects consist of several parts.
- Some of these parts depend on others, in the sense that we cannot design one part until the other part is ready.
- For example, many numerical techniques for solving systems of *nonlinear* equations use linearization.
- Thus, they solve systems of *linear* equations at different stages; so:
  - in order to design a part for solving systems of nonlinear equations,
  - we need to have available a part for solving systems of linear equations.
- This dependency relation makes the set of all parts into a partially ordered set.

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## 5. Formulating the Problem in Precise Terms (cont-d)

- In other words, this set a directed acyclic graph.
- For each part  $i$ , we know the overall effort  $e_i$  (e.g., in man-hours) that is needed to design this part.
- We also know the profit  $p_i$  that we can start earning once this part is released; so:
  - if we release part  $i$  at time  $t_i$ ,
  - then by some future moment of time  $T$ , selling this part will bring us the profit of  $p_i \cdot (T - t_i)$ .
- How can we organize our work on different parts so as to maximize the resulting overall profit.

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## 6. What Is Known and What We Do

- Several semi-heuristic strategies are known that lead to optimal (or at least close-to-optimal) release schedules.
- Interestingly:
  - while it is, in principle, possible for the company to work on several parts at a time,
  - in all known optimal schedules, the design is performed one part at a time.
- We show that the above empirical fact is not a coincidence: we will actually prove that
  - in the optimal schedule,
  - we *should* always work on one part at a time.

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## 7. Main Result: Formulation and Discussion

- *For each planning problem, there is an optimal schedule in which we always work on one part at a time.*
- This result does not necessarily means that in *all* optimal scheduled, we work on one part at a time.
- Let us give a simple example.
- Suppose that the software consists of three parts:
  - The first two parts are independent and require the same time  $t_1 = t_2$ .
  - The third part depend on the first two parts.
- Suppose also that there is no market need for Part 1 or for Part 2, only for the final Part 3.
- In other words, we assume that  $p_1 = p_2 = 0$ .

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## 8. Discussion (cont-d)

- In this case, it is reasonable to conclude that the following schedule is optimal:
  - first, we work on Part 1; this take time  $t_1$ ;
  - then, we concentrate all or efforts on Part 2; this also takes time  $t_1 = t_2$ ;
  - after this, we work on Part 3; this takes time  $t_3$ .
- So, by time  $2t_1 + t_3$ , we get the product that we can start selling.
- Alternatively, we can use a different schedule.
- First, we split the team into two equal sub-teams, with
  - one sub-team working on Part 1, and
  - the other sub-team working on Part 2.
- Designing each part takes time  $t_1 = t_2$  for the whole team.

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## 9. Discussion (cont-d)

- Thus, it will take twice longer for the twice-smaller sub-teams.
- After the time  $2t_1$ , both Part 1 and Part 2 are ready, so we can start working on Part 3.
- At the end, after time  $2t_1 + t_3$ , we will get the ready-to-sell product.
- In this example, at least one of the parts does not bring any profit, i.e., has  $p_i = 0$ . We will see that
  - if each part can bring some profit, i.e., if  $p_i > 0$  for all  $i$ ,
  - then such examples are not possible, and in *all* optimal schedules, we work on one part at a time.

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## 10. Proof: Transformation and Its Properties

- Let us start with an optimal schedule in which
  - at some moment of time,
  - we work on several parts at the same time.
- Let  $t_f$  be the first moment of time with this property.
  - If any part which is ready by this moment is not yet released in the original optimal plan,
  - then we can release it right away and thus provide the additional income from selling this part.
- Thus, without losing generality, we can assume that:
  - all the parts which are released after moment  $t_f$
  - are not yet fully ready for release at the moment  $t_f$ .
- Let  $t_r$  be the first moment of time after  $t_f$  at which one of the parts is being released.

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## 11. Transformation and Its Properties (cont-d)

- Let  $j$  be the number of the part that is released at moment  $t_r$ .
- Then, we can modify the original schedule as follows.
- Right after the moment  $t_f$ , we concentrate all our efforts on this Part  $j$ .
- All the efforts aimed at other parts are performed after that.
- By the time  $t_r$ , we spend the exact amount of efforts on all the parts as before.
- In the original plan, around moment  $t_f$ , we were also working on some other parts.
- So, this concentration means that we can now release Part  $j$  earlier.

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## 12. Transformation and Its Properties (cont-d)

- However, we preserve the release dates for all other parts.
- Thus, in comparison with the original plan:
  - we can earn more (or at least same amount of) profit,
  - since we started selling Part  $j$  earlier.

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## 13. Two Possibilities

- The profit  $p_j$  is always non-negative.
- So, we have two options:
  - $p_j > 0$  and
  - $p_j = 0$ .
- Let us consider these two options one by one.

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## 14. Case When $p_j > 0$

- If  $p_j > 0$ , we indeed get more profit.
- This contradicts to our assumption that the original plan was optimal.
- Thus, when  $p_i > 0$  for all parts, then
  - in the optimal plan,
  - we cannot have moments of time at which we work on several parts at a time.
- This is exactly what we are trying to prove.

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## 15. Case When $p_j = 0$

- If  $p_j = 0$ , then we do not get any contradiction.
- We do get a new optimal plan in which:
  - either we always work on one part at a time
  - or the first moment of time at which we work on several parts at a time moves further,
  - to some moment  $t'_f > t_f$ ,
  - since in the vicinity of the moment  $t_f$ , we now work on only one part.

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## 16. We Have at Least One Fewer Part on Which We Work Simultaneously with Others

- Note that in the new plan, when we work on Part  $j$ , we always work *only* on this part.
- Indeed,  $t_f$  is the first moment at which we work on several parts at a time.
- Thus, this property has been true – and remains true – before the moment  $t_f$ .
- We have also designed a new plan in such a way that this is also true for all moments  $t \geq t_f$ .

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## 17. If the Resulting Plan Is Not The Desired One, We Can Apply Similar Transformations

- In the new plan, we still have moments of time when we simultaneously work on several parts.
- Then, by applying the same transformation to the new first-such-moment  $t'_f$ , we get yet another optimal plan.
- For this new plan:
  - either there is no time when we several parts at a time,
  - or the first moment  $t''_f$  when this happens is even further away.

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## 18. Let Us Prove That This Process Converges to the Desired Plan

- At each such step:
  - from the list of parts on which we work simultaneously with others,
  - we delete at least one part.
- Thus, after at most as many steps as there are parts, we will get an optimal plans in which:
  - no such parts remain and thus
  - we always work on one part at a time.
- This final optimal plan is exactly the plan whose existence we wanted to prove.
- Thus, our main result is proven.

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## 19. Acknowledgments

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