Fuzzy Approach to Optimal Placement of Health Centers

Juan Carlos Figueroa Garcia[1], Carlos Franco[2], and Vladik Kreinovich[3]

[1]Department of Industrial Engineering
Universidad Distrital, Bogotá D.C, Colombia, filthed@gmail.com
[2]School of Management, Universidad del Rosario
Bogotá, Colombia, carlosa.franco@urosario.edu.co
[3]Department of Computer Science, University of Texas at El Paso
El Paso, Texas 79968, USA, vladik@utep.edu

1. Need for Health Centers

- Many countries in the world have socialized medicine – in this sense, US is one of the few exceptions.
- In such countries, it is important to decide how to distribute the limited resources.
- The objective is to best serve the population.
- In some case, all the patient needs is a regular general doctor; however, in many other cases:
  - the patient also needs to undergo some tests – blood test, X-ray, etc.,
  - he/she may need to see a specialist, etc.
- It is more convenient for the patients if all the need medical professionals are placed at a single location.
- This is the main idea behind health centers.

2. Where to Place Health Centers?

- Where are the best locations for these centers?
- And, once we find these locations, what is the best way to assign each patient to one of these centers?
- These are the problems that were raised in our previous paper.
- These are the problems that we deal with in this paper as well.

3. What We Know

- Let X denote the area that we want to serve with health centers.
- Let $\rho(x)$ denote the population density at geographic location x, i.e., the number of people per unit area.
- Once we know the population density, we can compute the overall number of people in any given area A as

$$\int_A \rho(x)\,dx.$$

- Let $P = \int_X \rho(x)\,dx$ denote the overall number of people in our area X.

4. What We Want

- We need to decide how many health centers to place at different locations.
- Let $h(x)$ denote the number of health centers per unit area in the vicinity of a geographical location x.
- Once we determine this density $h(x)$, we can compute the overall number of health centers in area A as

$$\int_A h(x)\, dx.$$

5. Main Limitation and Objective Function

- Our resources are limited: we can only build so many health centers.
- Let N denote the overall number of health centers that we can build, then $\int_X h(x)\,dx = N$.
- In the ideal world, every patient should be immediately seen by a doctor.
- In reality, it takes some time for a patient to reach the nearest health center.
- The smaller this time, the better.
- Thus, a reasonable objective function is the average time that it takes for a patient to reach a doctor.
- Let us describe this objective function in precise terms.

6. Objective Function: Towards a Formal Description

- The time $t(x)$ that it takes for a patient at location x to reach the nearest health center can be computed as

$$t(x) = \frac{d(x)}{v(x)}, \text{ where:}$$

  - $d(x)$ is the distance from location x to the nearest health center, and
  - $v(x)$ is the average transportation speed in the vicinity of the location x.

- The speed $v(x)$ is usually:
  - smaller in the city center,
  - slightly larger in the suburbs, and
  - even larger outside the city limits.

7. Objective Function (cont-d)

- Let $m(x)$ denote the maximum distance $m(x)$ that it takes for points around x to reach a doctor.
- This distance corresponds to the case when the location is at the edge of the zone allocated to this center.
- So, it is attained at the edge of a disk of radius $m(x)$ served by this center.
- In this circle, there is exactly one health center.
- Based on the density $h(x)$ of health centers, we can estimate the number of health centers in this disk area:

$$\int h(x)\,dx \approx h(x)\cdot(\pi\cdot m(x)^2).$$

- We know that this value is 1, since there is only one health center in this disk area.

8. Objective Function (cont-d)

- So, we conclude that $h(x)\cdot(\pi\cdot m(x)^2)=1$, i.e., that

$$m(x)=\frac{1}{\sqrt{\pi\cdot h(x)}}.$$

- What is the average distance $d(x)$ from a center of the disk of radius $m(x)$ to a point on this disk?
- For all the points at distance r from the center, this distance is r.
- The area of the small vicinity of this disk is $2\pi\cdot t\,dr$.
- Thus, the average distance can be computed as

$$\frac{1}{\pi\cdot(m(x))^2}\cdot\int_0^{m(x)} r\cdot(2\pi\cdot r\,dr)=\frac{1}{\pi\cdot(m(x))^2}\cdot\frac{2}{3}\cdot\pi\cdot(m(x))^2=$$

$$\frac{2}{3}\cdot m(x),\text{ so } d(x)=\frac{2}{3\cdot\sqrt{\pi}}\cdot\frac{1}{\sqrt{h(x)}}.$$

9. Objective Function (cont-d)

- So, $t(x)=\dfrac{d(x)}{v(x)}=\dfrac{2}{3\cdot\sqrt{\pi}}\cdot\dfrac{1}{\sqrt{h(x)}\cdot v(x)}$.
- This is the time that it takes for each patient to reach the health center.
- The average time that it takes all the patients to reach the health center can be then computed as

$$\frac{1}{P}\cdot\int_X \rho(x)\cdot t(x)\,dx=\frac{2}{3\cdot\sqrt{\pi}\cdot P}\cdot\int_X \frac{\rho(x)}{\sqrt{h(x)}\cdot v(x)}\,dx.$$

- Now, we are ready to formulate the problem in precise terms.

10. Exact Formulation of the Problem and Its Solution

- We know the functions $\rho(x)$ and $v(x)$.
- Based on this knowledge, we need to find the function $h(x)$ that, under constraint $\int h(x)\,dx = P$, minimizes
$$\frac{2}{3\cdot\sqrt{\pi}\cdot P}\cdot\int_X \frac{\rho(x)}{\sqrt{h(x)}\cdot v(x)}\,dx.$$
- Multiplying all the value of the objective function by the same constant does not change which value is larger.
- Thus, minimizing the above objective function is equivalent to minimizing a simpler expression
$$\int_X \frac{\rho(x)}{\sqrt{h(x)}\cdot v(x)}.$$
- To solve this problem, we can use the Lagrange multiplier method.

11. Solving the Problem (cont-d)

- So, our constraint optimization problem is equivalent, for some λ, to the unconstrained problem of minimizing

$$\int_X \frac{\rho(x)}{\sqrt{h(x)} \cdot v(x)} + \lambda \cdot \left(\int_X h(x)\, dx - N \right).$$

- Differentiating this expression with respect to the unknown $h(x)$ and equating the derivative to 0, we get

$$-\frac{1}{2} \cdot \frac{\rho(x)}{(h(x))^{2/3} \cdot v(x)} + \lambda = 0.$$

- So, $h(x) = c \cdot \left(\frac{\rho(x)}{v(x)} \right)^{2/3}$, for some constant c.

12. Solving the Problem (cont-d)

- The constant c can be found if we substitute the above expression into the constraint; then, we get

$$h(x) = N \cdot \frac{\left(\frac{\rho(x)}{v(x)}\right)^{2/3}}{\int_X \left(\frac{\rho(y)}{v(y)}\right)^{2/3} dy}.$$

13. Discussion

- The density of health centers is proportional to the population density raised to the power $2/3$.
- Thus, in the regions with higher population density $\rho(x)$, we place more health centers.
- However, the number of health centers grows slower than the population density.

14. How Many Doctors Are Needed in Each Health Center

- What us the number of medical personnel $M(x)$ needed for each health center?
- It is proportional to the number of people $N(x)$ served by each center: $M(x) = m_0 \cdot N(x)$.
- The coefficient m_0 can be obtained if we know the overall number M of medical professionals.
- Indeed, for the whole population, the above formula implies that $M = m_0 \cdot P$; thus, $m_0 = \dfrac{M}{P}$ and

$$M(x) = \frac{M}{P} \cdot N(x).$$

15. How Many Doctors Are Needed (cont-d)

- The number of people $N(x)$ served by a health center can be obtained by multiplying:
  - the population density $\rho(x)$ in the vicinity of a given location x
  - by the area $1/h(x)$ covered by the center:

$$M(x) = \frac{M}{P} \cdot \frac{\rho(x)}{h(x)}.$$

- We know $h(x)$, hence

$$M(x) = \frac{M}{P \cdot N} \cdot \left(\int_X \left(\frac{\rho(y)}{v(y)} \right)^{2/3} dy \right) \cdot (\rho(x))^{1/3} \cdot (v(x))^{2/3}.$$

16. Where to Actually Place the Health Centers?

- The above formulas describe how many health centers to place in the vicinity of each location x.
- But where exactly should we place them?
- We need to find 2-D locations $p_1, \ldots, p_N$ so that the average distance to a center be the smallest possible.
- For each location x, let us denote, by $i(x)$, the number of the health center associated with this location.
- Then, the distance from each location x to the corresponding health center is $d(x, p_{i(x)})$.
- The travel time is equal to $\dfrac{d(x, p_{i(x)})}{v(x)}$.
- The average distance can be therefore computed as

$$\int \rho(x) \cdot \frac{d(x, p_{i(x)})}{v(x)}\, dx.$$

17. Where to Place the Health Centers (cont-d)

- If we take into account the discrete character of the information, we get the sum

$$\sum_x \rho(x) \cdot \frac{d(x, p_{i(x)})}{v(x)}.$$

- We need to find the values $p_1, \ldots, p_N$ and the value $i(x)$ (for all $x \in X$) that minimize this expression.

18. Towards an Algorithm

- It is difficult to immediately minimize the above objective function with respect to all the unknowns.
- So a natural idea is to minimize it iteratively.
- Namely, we start with some location of the centers.
- Then, we fix the locations p_i of the health centers and find the corresponding assignments $i(x)$.
- For each location x, this means minimizing the distance $d(x, i(x))$, i.e., finding the health center closest to x.
- Then, we find the locations p_i that, for these assignments $i(x)$, minimize the objective function.
- For each health center i, this is equivalent to finding the new location p_i that minimizes the average distance

$$\sum_{x:i(x)=i} \rho(x) \cdot \frac{d(x, p_i)}{v(x)}.$$

19. Towards an Algorithm (cont-d)

- This can be done, e.g., by gradient descent.
- Then, we repeat the procedure again and again until the process converges.
- This means that locations on previous and next iteration are ε-close, for some ε.
- This is similar to the standard algorithm for computing fuzzy clusters, where we iteratively:
  - first, assign each point to clusters depending on this point's distance to the cluster centers, and
  - then find the new centers which are, on average, closest to all the points assigned to the cluster.

20. Resulting Algorithm

- We first randomly place the centers in accordance with the center density $h(x)$.
- Then, we iteratively do the following:
- First, for each spatial location x, we find the closest health center.
- We will denote the index of this health center by $i(x)$.
- Then, for each i from 1 to N, we find a new location p_i that minimizes the average distance.
- This is done, e.g., by gradient descent.
- Then, we repeat the procedure again and again until the process converges.
- This means that for locations p_i and p'_i on two consequent iterations $d(p_i, p'_i) \le \varepsilon$ for all i.

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21. Need for a Fuzzy Approach

- In the above description, we assumed that each location is assigned to exactly one health center.
- This assignment was based on the simplified assumption that the travel time is deterministic.
- In reality, as everyone who lives in a big city knows, travel time can change drastically.
- Sometimes there are traffic jams, sometimes there are accidents.
- Also, we only took into account travel time, but there is also waiting time.

22. Need for a Fuzzy Approach (cont-d)

- From this viewpoint:
  - if we have a patient who is slightly closer to one health venter than to the other,
  - it does not make sense to assign this patient always to the nearest health center.
- Maybe there is a long waiting time in the nearest health center, but no waiting time in another.
- As a result, the patient will be served faster if he or she goes to this second health center this time.
- Instead of assigning each patient to a single health center, it is beneficial to make a "fuzzy" allocation.
- We allow the patient to go to any health center in the nearest vicinity.

23. Need for a Fuzzy Approach (cont-d)

- The patient should go to the doctor for which the travel time + waiting time is the smallest.
- There are many apps already for predicting travel time.
- There are similar apps for predicting the waiting time.
- Nowadays, most medical records are electronic.
- So, it is not a problem to access the records from each of the health centers.

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