How Mathematics and Computing Can Help Fight the Pandemic: Two Pedagogical Examples

Julio C. Urenda, Olga Kosheleva
Martine Ceberio, and Vladik Kreinovich

University of Texas at El Paso
El Paso, TX 79968, USA
jcurenda@utep.edu, olgak@utep.edu
mceberio@utep.edu, vladik@utep.edu

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

1. First Example: Need for Social Distancing

- This problem is related to the pandemic-related need to observe a social distance of at least 2 meters (6 feet).
- Two persons are on two sides of a narrow-walkway street, waiting for the green light.
- They start walking from both sides simultaneously.
- For simplicity, let us assume that they walk with the same speed.
- If they follow the shortest distance path – i.e., a straight line AB – they will meet in the middle.
- This is not good; so one of them should move somewhat to the left, another should move somewhat to the right.
- At all moments of time, they should be at least 2 meters away from each other.
- What is the fastest way for them to do it?

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

2. Formulating This Problem in Precise Terms

- The situation is absolutely symmetric with respect to the reflection in the midpoint M of the segment AB.
- So, it is reasonable to require that:
  - the trajectory of the 2nd person can be obtained from the trajectory of the 1st person
  - by this reflection.
- Thus, at any given moment of time, the midpoint M is the midpoint between the two persons.
- In these terms:
  - the requirement that they are separated by $\geq 2m$
  - means that each of them should always be at a distance at least 1 meter from the midpoint M.
- In other words, both trajectories should avoid the disk of radius 1 meter with a center at the midpoint M.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

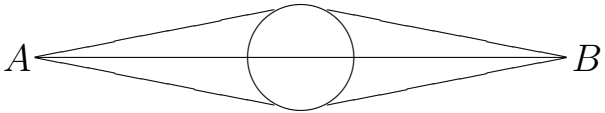
3. Formulating the Problem (cont-d)

- We want the fastest possible trajectory.
- The speed is assumed to be constant.
- So, they should follow the shortest possible trajectory.
- In other words, we need to find:
  - the shortest possible trajectory going from point A to point B
  - that avoids the disk centered at the midpoint M of the segment AB.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

4. Solution

- To get the shortest path, outside the disk, the trajectory should be straight.
- Where it touches the circle, it should be smooth.
- Thus, the solution is as follows:
  - first, we follow a straight line until it touches the circle as a tangent,
  - then, we follow the circle,
  - and finally, we follow the straight line again – which again starts as a tangent to the circle:



First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

5. Second Example: Need for Fast Testing

- One of the challenges related to the COVID-19 pandemic is that:
  - this disease has an unusually long incubation period,
  - about 2 weeks.
- As a result, people with no symptoms may be carrying the virus and infecting others.
- As of now, the only way to prevent such infection is to perform massive testing of the population.
- The problem is that there is not enough test kits to test everyone.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

6. What Was Proposed

- To solve this problem, researchers proposed the following idea:
  - instead of testing everyone individually,
  - why not combine material from a group of several people, and
  - test each combined sample by using a single test kit.
- If no viruses are detected in the combined sample, this means that all the people from the corresponding group are virus-free.
- So there is no need to test them again.
- After this, we need to individually test only folks from the groups that showed the presence of the virus.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

7. Resulting Problem

- Suppose that we need to test a large population of N people.
- Based on the previous testing, we know the proportion p of those who have the virus.
- In accordance with the above idea, we divide N people into groups.
- The question is: what should be the size s of each group?
- If the size is too small, we are still using too many test kits.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

8. Resulting Problem (cont-d)

- If the size is too big, then:
  - every group, with a high probability, has a sick person,
  - so we are not dismissing any people after such testing, and thus, we are not saving any kits at all.
- So what is the optimal size of the group?
- Of course, this is a simplified formulation.
- It does not take into account that:
  - for large group sizes s, when each individual testing material is diluted too much,
  - tests may not be able to detect infected individuals.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

9. Let Us Formulate This Problem in Precise Terms

- If we divide N people into groups of s persons each, we thus get N/s groups.
- The probability that a person is virus-free is $1 - p$.
- Thus, the probability that all s people from a group are virus-free is $(1 - p)^s$.
- So, out of N/s groups, the number of virus-free groups is $(1 - p)^s \cdot (N/s)$.
- Each of these groups has s people.
- So the overall number of tested people can be obtained by multiplying the number of virus-free groups by s.
- This results in $(1 - p)^s \cdot N$.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

10. Formulation and Solution

- For the remaining $N - (1-p)^s \cdot N$ folks, we need individual testing.
- So, the overall number of needed test kits is

$$N_t = \frac{N}{s} + N - (1-p)^s \cdot N.$$

- We want to minimize the number of test kits.
- So, we want to find the group size s for which this number is the smallest possible.
- Equating the derivative to 0 and dividing both sides of this equation by N, we get:

$$-\frac{1}{s^2} - (1-p)^s \cdot \ln(1-p) = 0.$$

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

11. Solution (cont-d)

- For small p, we have $(1-p)^s \approx 1$ and $\ln(1-p) \approx -p$, so $-\dfrac{1}{s^2} + p \approx 0$, and $s \approx \dfrac{1}{\sqrt{p}}$.
- For example:
  - for $p = 1\%$, we have $s \approx 10$;
  - for $p = 0.1\%$, we get $s \approx 30$; and
  - for $p = 0.01\%$, we get $s \approx 100$.
- The resulting number of tests N_t can also be approximately estimated.
- When the group size s is described by the approximate formula, we have $\dfrac{N}{s} \approx \sqrt{p} \cdot N$.
- If we take into account that $(1-p)^s \approx 1 - p \cdot s$, then

$$N - (1-p)^s \cdot N \approx p \cdot s \cdot N \approx \sqrt{p} \cdot N.$$

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

12. Solution (cont-d)

- Thus, we get $N_t \approx \sqrt{p} \cdot N$.
- For example:
  - for $p = 1\%$, we need 10 times fewer test kits than for individual testing;
  - for $p = 0.1\%$, we need 30 times fewer test kits; and
  - for $p = 0.01\%$, we need 100 times fewer test kits.

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)

13. Bibliography

- T. S. Perry, "Researchers are using algorithms to tackle the coronavirus test shortage: the scramble to develop new test kits that deliver faster results", *IEEE Spectrum*, 2020, Vol. 57, No. 6, p. 4.
- N. Shental, S. Levy, V. Wuvshet, S. Skorniakov, Y. Shemer-Avni, A. Porgador, and T. Hertz, *Efficient High Throughput SARS-CoV-2 Testing to Detect Asymptomatic Carriers*, medRxiv preprint https://doi.org/10.1101/2020.04.14.20064618, posted on April 20, 2020.

14. Acknowledgments

This work was supported in part by the National Science Foundation grants:

- 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science),
- HRD-1242122 (Cyber-ShARE Center of Excellence).

First Example: Need . . .
Formulating This . . .
Solution
Second Example: . . .
What Was Proposed
Resulting Problem
Let Us Formulate This . . .
Formulation and Solution
Solution (cont-d)