

Why Base-20, Base-40, and Base-60 Number Systems?

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Historical Facts

But Why?

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1. Historical Facts

- In the ancient times:
 - in addition to our usual base-10 number system and to systems with a smaller or similar-size base,
 - some cultures used number systems with much larger bases.
- Babylonians used the 60-based system.
- We still divide an hour into 60 minutes, a minute into 60 seconds.
- This idea originated with the ancient Babylonians.
- Ancient Romans used the base-20 system.

2. Historical Facts

- This can still be traced to how numbers are named in modern French; for example:
 - in French, 80 is quatre-vingts, meaning four-twenties, and
 - in French, 96 is quatre-vingt-seize, meaning four-twenties-sixteen.
- A similar 20-based system – with 20 divided into four 5s – was used by the Mayans and by the Aztecs
- An unusual 40-based system was used in medieval Russia; for example:
 - to describe the (large) number of churches in the medieval Moscow,
 - the Russian chronicle says that there were 40 of 40s (sorok sorokov), i.e., $40 \cdot 40 = 1600$.

3. But Why?

- A natural question is: why these bases and not others?
- There is a good explanation of why 60:
- This is the number that has unusually many divisors: it is divisible by 2, 3, 4, 5, 6, 10, 12, 15, and 20; so:
 - $1/3$ of a usual 60-minute hour is a whole number of minutes,
 - $1/4$ of an hour is a whole number of minutes, etc.
- This would not have been possible if we divided an hour into 100 minutes.
- There is a similar partial explanation of base 20.
- However, there is no similar explanation for selecting 40.

4. But Why (cont-d)

- Moreover:
 - from the viewpoint of the above explanation of the base-60 system,
 - the values 20 and 40 are not good.
- For example, if the Romans selected 24 or 30 instead of 20, they would have had many more divisors.
- In this talk, we provide a possible explanation for all three number bases.
- This explanation is based on analyzing practical problems that ancient and medieval folks faced.

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5. Practical Problem: Management

- Ancient and medieval civilizations had many activities involving large groups of people.
- One of the main reasons why civilizations appeared in the first place was the possibility of big projects:
 - big irrigation projects,
 - building a protective fortress,
 - having a strong army to make peaceful life possible, etc.
- When you have a large group of people involved in a certain activity, it is important to manage them.
- This problem is not as acute in the army.
- Indeed, soldiers are trained to follow orders – and thus, to be managed.

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6. Practical Problem: Management (cont-d)

- However, effective management is crucial in civilian projects.
- Indeed, in such projects, most workers do not have special training in following orders.
- These workers need to be organized.
- There is also a need to have managers (overseers) for overseeing the organized groups of workers.
- When the overall number of workers is very large, it is not enough to simply organize workers in groups:
- There will still be too many groups.
- So we need to combine groups into groups of higher level.
- In other words, we need to have a hierarchical organization.

7. Let Us Start at the Lowest Level of the Hierarchy

- On the lowest level of the hierarchy, we need to combine workers into working groups.
- How many people can one boss effectively oversee?
- To answer this question, we need to take into account that:
 - according to psychology, there is a “seven plus minus two” law,
 - according to which a person can only keep between $7 - 2 = 5$ and $7 + 2 = 9$ ideas in mind.
- How many depends on the person:
 - some can only keep 5,
 - some can keep 9.

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8. Lowest Level (cont-d)

- We want to make sure that any person can serve as a supervisor of such lower-level group.
- So, we need to make sure that this group contains no more than 5 people; otherwise:
 - people who can only keep 5 ideas in their mind at the same time
 - will not be able to effectively supervise this group.
- On the other hand, everyone can keep 5 ideas.
- So it will be a waste of resources to make these primary groups with fewer than 5 folks.
- Thus, the ideal size of the primary group is 5.

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9. Lowest Level (cont-d)

- This argument shows that it is reasonable to expect base-5 number systems.
- Such systems have actually been used by several cultures.

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10. Second Level of the Hierarchy

- Even if we divide thousands of workers into groups of 5, we will get many groups.
- So, to effectively supervise these primary groups, we need to combine them into secondary groups.
- How many primary groups should we combine into a secondary one?
- It is much more difficult to be a boss of bosses than simply a low-level boss of people.
- Each lower-level group has 5 folks.
- So, the number of primary groups combined into a secondary group should be smaller than 5.
- So, we have three options.
- We can have 4 groups of 5, making up 20.

11. Second Level of the Hierarchy (cont-d)

- This explains the base-20 system.
- Actually, the Mayans explicitly considered 20 as 4 groups of 5.
- We can have 3 groups of 5, making up 15.
- Historically, there is no direct evidence of base-15 systems.
- However, there is an indirect evidence.
- For example, Russia used to have 15-kopeck coins, a very unusual nomination.
- We can have 2 groups of 5, making up 10; this is our usual decimal system.
- Its representation as two groups of 5 can be seen, e.g., in the design of the abacus.

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12. Third Level

- On the next level, it is even more difficult to manage.
- So the number of secondary groups that form a ternary group must be smaller than the number of primary groups in a secondary group.
- For $10 = 2 \cdot 5$, there is no possibility to have fewer than 2 secondary groups.
- For $15 = 3 \cdot 5$, the only option is having 2 groups of 15 together, making it $2 \cdot 15 = 30$.
- There does not seem to be any evidence of any culture using base-30 number systems.

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13. Third Level (cont-d)

- For $20 = 4 \cdot 5$, we have two options:
 - having 3 groups of 20, making it $3 \cdot 20 = 60$; and
 - having 2 groups of 20, making it $2 \cdot 20 = 40$.
- The last two options provide an explanation of why 60 and 40 were used as bases.

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14. Why 60 in Babylon, 40 in Russia, and 20 in Europe: Brainstorming

- The above arguments explain why 20, 40, and 60 were used as bases.
- However, they do not explain why different systems appeared in different countries.
- This requires going beyond mathematics, to history.
- We are not historians, but we can try to speculate.
- Our speculation is based on the natural idea that:
 - the more obedient people are, the less they rebel, the easier it is to control them, and
 - thus, the larger ternary groups can be formed.

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15. Brainstorming (cont-d)

- Babylonia was ruled by mighty rulers for several centuries.
- So, it could perform a control of the largest number of 20-size groups supervised by one person: 3.
- This explains why the corresponding value $3 \cdot 20 = 60$ was used in Babylonia.
- Medieval Russia was also ruled with a heavy hand, but there were still many riots and uprisings.
- So, it could afford only the smaller number of 20-size groups supervised by one person: 2.
- This explains why the corresponding value $2 \cdot 20 = 40$ was used in Russia.

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16. Brainstorming (cont-d)

- Finally, the Roman Empire was the site of many uprisings and revolts.
- This kind of explains why even combining two 20-size groups under one person was difficult.
- This is why the ancient Romans only used base-20 system.

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