

Dimension Compactification Naturally Follows from First Principles

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1. What is dimension compactification

- According to modern physics:
 - the requirement that the quantum field theory be consistent
 - implies that the dimension of space-time should be at least 11.
- How can we combine this conclusion with the fact that the observed space-time is only 4-dimensional?
- A usual explanation is that:
 - while in the beginning, space-time may have had 11 or more equally prominent dimensions,
 - with time, most of these dimensions has been *compactified*.
- To be more precise:
 - the size in the direction of these additional dimension remains as small as the Universe was in its first moments, while
 - other dimensions expanded to the current astronomical sizes.

2. Compactification: how and why

- There are several mechanisms that explain *how* compactification could have happened.
- However, these mechanisms do not explain *why* it happened.
- In this talk, we provide arguments that compactification naturally follows from first principle.
- We actually provide *two* first-principles explanations for space-time compactification:
 - an explanation based on the Second Law of Thermodynamics and
 - an explanation based on geometry and symmetries.

3. Second Law of Thermodynamics: a brief reminder

- According to the Second Law of Thermodynamics:
 - the entropy of the Universe (and of any closed system)
 - increases with time (or, in some cases, stays the same).
- There is no limit to such increase, eventually we get closer and closer to the state with the largest possible entropy.
- In general, the entropy is defined as $S = - \int \rho(x) \cdot \ln(\rho(x)) dx..$
- Here, $\rho(x)$ is the probability distribution of the set of all possible micro-states.

4. How is entropy depending on dimension

- In general:
 - close points or close particles are strongly correlated, while
 - distant particles are independent.
- A simplified description of this phenomenon can be obtained if we assume that all the points are divided into groups of nearby ones, so that:
 - within each group there is a correlation, but
 - between the groups there is no correlation.
- It is known that:
 - if we have several independent random processes,
 - then the overall entropy is equal to the sum of the entropies of these processes.

5. How is entropy depending on dimension (cont-d)

- Thus, to find the overall entropy of the Universe in this approximation, it is sufficient:
 - to compute the entropy corresponding to each group, and then
 - to add up the resulting entropies.
- How many points n are in each such group?
- Let us consider first the case:
 - when we only consider immediate neighbors,
 - i.e., points whose all coordinates different from this one by no more than 1 appropriate unit of distance.
- Let's consider a coordinate system in which a central particle is at the point $(0, \dots, 0)$.
- Then, each of d coordinates of an immediate neighbor is equal to -1 , 0 , or 1 – three options.

6. How is entropy depending on dimension (cont-d)

- So overall, we have $n = 3^d$ points.
- If we consider neighbors of neighbors, we can have 5^d points – and, in general, $n = a^d$ for some $a > 1$.
- This number clearly grows with the dimension d .
- So, when we go from a higher dimension d to a lower dimension $d' < d$, the number of neighbors decreases.
- This means that:
 - instead of the original group of size n in which all particles were correlated,
 - we have several subgroups of smaller size, and there is no longer correlation between different subgroups.

7. How is entropy depending on dimension (cont-d)

- It is known that:
 - if we know distributions corresponding to all the subgroups,
 - then the entropy of the overall distribution for the whole group is the largest if and only if these subgroups are independent.
- Thus, when we divide a group in which all elements were correlated into smaller independent subgroups, we increase entropy.
- According to the usual interpretation of the Second Law of Thermodynamics, there are no limitations to the increase in entropy.
- So, eventually, we should also encounter a decrease in spatial dimension as a way to increase entropy.
- This is exactly what compactification is about.

8. Comment

- The above argument does not imply that compactification will stop at our 3 dimensions.
- It can go further, to having a 2- and even 1-dimensional space.
- Maybe this is what is already happening in the Universe, with 1D superclusters of Galaxies.

9. Explanation Based on Geometry and Symmetries

- The original distribution of matter was uniform.
- However, the uniform distribution is not stable:
 - if at some point, due to fluctuations, the density becomes larger than at the neighboring points,
 - then this point start attracting matter from its neighbors – thus further increasing its density.
- As a result, you get a large disturbance.

10. Symmetries and statistical physics: general idea

- The original distribution in a d -dimensional space was invariant under shifts, rotations, and scaling (i.e., transformation $x_i \rightarrow \lambda \cdot x_i$).
- According to statistical physics:
 - It is not very probably that from a highly symmetric state, we go straight into a completely asymmetric one.
 - Usually, the most probably transition is to a state that preserves as many symmetries as possible.
- So, we expect the shapes of the disturbances to have some symmetries.

11. Analysis of the problem

- What is the shape:
 - that has the largest number of symmetries,
 - i.e., for which the dimension of the corresponding symmetry group is the largest?
- If the shape is invariant with respect to all rotations in the d -dimensional space, then it must consist of spheres.
- A sphere has only rotations – so the dimension of the corresponding symmetry group is $\frac{d \cdot (d - 1)}{2}$.
- Indeed, infinitesimal rotations are described by asymmetric matrices which have exactly as many parameters.
- So, in this case, the dimension of the symmetry group is $\frac{d^2 - d}{2}$.

12. Analysis of the problem (cont-d)

- If the shape includes a $(d - 1)$ -dimensional space, then we have:
 - $d - 1$ independent shifts,
 - $\frac{(d - 1) \cdot (d - 2)}{2}$ independent rotations, and
 - 1 scaling,
 - to the total of

$$d - 1 + \frac{(d - 1) \cdot (d - 2)}{2} + 1 = \frac{d^2 - d + 2}{2}.$$

- This is larger than for the sphere.
- If we have all $(d - 1)$ -dimensional rotations but not all shifts or scaling, then we have fewer symmetries.
- What if we only have rotations in a $(d - 2)$ -dimensional space, to the total of $\frac{(d - 2) \cdot (d - 3)}{2}$?

13. Analysis of the problem (cont-d)

- We cannot have $d - 1$ shifts, because this would lead to a $(d - 1)$ -dimensional space.
- Thus, we can have no more than $d - 2$ independent shifts.
- Even if we have $d - 2$ shifts and rotations, we will have

$$d - 2 + \frac{(d - 2) \cdot (d - 3)}{2} + 1 \text{ independent symmetries.}$$

- This is smaller than $d - 1 + \frac{(d - 1) \cdot (d - 2)}{2} + 1$.
- Conclusion:
 - the most probable result of a natural spontaneous symmetry violation of a d -dimensional space
 - is a $(d - 1)$ -dimensional space.

14. Analysis of the problem (cont-d)

- Since fluctuations continue, we will then get space of dimension $d - 2$, etc.
- This provides another explanation of why the original space has lost many of its dimensions.

15. Comments

- We have two explanations of the same phenomenon.
- However, these explanations are not contradicting each other.
- Both are based on statistical physics, we just took into account different aspects of it.
- The above idea of shapes motivated by symmetries has been used in physics.
- For example, it explains the existing shapes of celestial bodies.

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