Is Earth's tilt a resonance?

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1. Earth tilt: its importance and its value

- The orbits of all the planets of the Solar system lie in the same plane.
- The rotation axes of all the planets are approximately orthogonal to this plane.
- However, each actual rotation axis is somewhat tilted in related to this orthogonal direction.
- This tilt is the reason why, in most places on Earth, we have four seasons:
  - Winter, when this location is tilted away from the Sun and thus, gets the smallest amount of the Solar radiation energy;
  - Spring, when the amount of Solar radiation increases;
  - Summer, when this located is tilted towards the Sun and thus, gets the largest amount of the Solar radional energy; and
  - Fall, when the amount of Solar radiation decreases.
- Specifically, the Earth's tilt is 23.4°.

2. An interesting fact about the Earth's tilt

- Sines and cosines of the angles appear in many formulas describing a system's dynamics.
- Interesting, the sine of the Earth's tilt angle is almost equal to 2/5:

$$\sin(23.4^\circ) = 0.397 \approx 0.4 = \frac{2}{5}.$$

3. Why this is an interesting fact – and the resulting question

- This fact is interesting because usually, unit-less combinations of physical quantities are generic real numbers.
- It is very rare that their value is close to a fraction of two small integers.
- In most cases when we have such an exceptional ratio, this value is caused by a *resonance*.
- Resonance is a physical phenomenon according to which two nearby pendulums (or other periodic processes) eventually get synchronized:
  - if their frequencies were close to each other, they become equal;
  - if one of the frequencies was almost twice larger than the second one, it becomes exactly twice larger, etc.

4. Why this is an interesting fact – and the resulting question (cont-d)

- For example, in the past:
  - the period of the Moon's rotation around the Earth was close to – but different from
  - the period of its rotation around its axis.
- However, with time, these periods became the same.
- So now, from the Earth, we can only see one side of the Moon.
- Resonance is not only about lifeless physical bodies, we can experience it ourselves.
- When the music is faster – e.g., when we hear dance music – our hearts start beating faster, and we become more active.

5. Why this is an interesting fact – and the resulting question (cont-d)

- On the other hand, when the music is slower – e.g., when we hear a lullaby:
  - our hearts start beating slower, and
  - we become less active – and may even fall asleep.
- So, the fact that we have a value close to $2/5$ leads to a natural question:
- Is this a resonance – or is it a random coincidence?
- In this paper, we try to answer this question.
- Our answer is: it is most probably not a resonance.

6. Towards a formal description of the problem

- In statistics:
  - practitioners usually conclude that an event is not a random coincidence
  - if the probability of this event happening randomly is smaller than a certain threshold.
- Usually this threshold is 0.05; sometimes, a smaller threshold is selected.
- To use this criterion, we need:
  - to formally describe the corresponding random situation – i.e., to come up with the appropriate probability distribution on the set of possible outcomes, and
  - to formally describe what we mean by the event.
- Let us do it.

7. Let us formally describe the random situation

- The outcome is the sine of the tilt angle.
- In principle, this angle can take any value between 0 and 90 degrees.
- So, the resulting sine can, in principle, take any value between 0 and!1.
- We have no a priori reason to think that some values from the interval $[0, 1]$ are more probable than others.
- It is therefore reasonable to assume:
  - that all the values from the interval $[0, 1]$ are equally probable,
  - i.e., that we have a uniform distribution on this interval.
- For this distribution:
  - the probability for a random value to belong to some subset – e.g., the union of disjoint intervals –
  - is equal to the overall length of this subset (i.e., to the sum of the lengths of its component intervals).

8. Let us formally describe the random situation (cont-d)

- We argued about having equal probabilities – if there is no reason to believe that one of the probabilities is larger.
- This argument goes back to Laplace, one of the founders of probability theory.
- It is known as *Laplace Indeterminacy Principle*.

9. Let us formally describe the event

- The event is that:
  - the sine of the tilt is 0.003-close to a ratio of two small natural numbers,
  - namely, of natural numbers not exceeding 5.
- What is the probability of this event?
- Let us list:
  - all the ratios of two natural numbers each of which is smaller than or equal to 5
  - for which the ratio is located between 0 and 1.
- With the denominator 1, we have $0/1 = 0$ and $1/1 = 1$.
- With the denominator 2, we have $0/2 = 0$, $1/2$, and $2/2 =1$.
- With the denominator 3, we have $0/3 = 0$, $1/3$, $2/3$, and $3/3 = 1$.

10. Let us formally describe the event (cont-d)

- With the denominator 4, we have $0/4 = 0$, $1/4$, $2/4 = 1/2$, $3/4$, and $4/4 = 1$.
- With the denominator 5, we have $0/5 = 0$, $1/5$, $2/5$, $3/5$, $4/5$, and $5/5$ =1.
- If we sort all these ratios in increasing order, we get the following list:

$$0 < 1/5 = 0.2 < 1/4 = 0.25 < 1/3 = 0.333\ldots < 2/5 = 0.4 < 1/2 =$$

$$0.5 < 3/5 = 0.5 < 2/3 = 0.888\ldots < 3/4 = 0.75 < 4/5 = 0.8 < 1.$$

- For each of the ratios 0 and 1, all possible 0.003-close real numbers form an interval of width 0.003.
- These intervals are, correspondingly, $[0, 0.003]$ and $[1 - 0.003, 1] = [0.997, 1]$.
- For each of the other ratios r, 0.003-close values form an interval $[r - 0.003, r + 0.003]$ of width 0.006.

11. Let us formally describe the event (cont-d)

- It is easy to see that the difference between every two consecutive numbers in the above list is larger than $2 \cdot 0.003 = 0.006$.
- Thus, all the corresponding intervals $[0, 0.003]$, $[r - 0.003, r + 0.003]$, and $[1 - 0.003, 1] = [0.997, 1]$ are disjoint.
- This mesnd that the intersection of any two of them is empty.
- Thus, the overall width of the union of these intervals is simply the sum of the widths of these intervals.
- We have nine intervals of width 0.006 each and two intervals of width 0.003 each.

12. Let us formally describe the event (cont-d)

- Thus, the overall width of the union of all these intervals is equal to $9 \cdot 0.006 + 2 \cdot 0.003 = 0.06$.
- So:
  - for the uniform distribution of the interval [0,1],
  - the probability that a random value is within one of these intervals is 0.06.

13. Conclusion: this is not a resonance, it is an accidental coincidence

- The probability that a random value is 0.003-close to one of the ratios of small natural numbers is equal to 0.06.
- It is, thus, larger than:
  - he usual threshold of 0.05,
  - the threshold that is normally used to reject the assumption that the value is random.
- And the value 0.06 is definitely larger than all possible smaller-than-0.05 thresholds that are sometimes used.
- Thus, we cannot conclude that the Earth's tilt is a resonance.

14. Comment

- Our Earth is just one of the eight planets.
- For no other planets, the sine of the tilt is 0.003-close to a ratio of two small natural numbers.
- What about other planets?
- For each planet, the probability that the random value is not that close is equal to $1 - 0.06 = 0.94$.
- Thus, the probability that the sine is not that close for all 8 planets is equal to $0.94^8 \approx 0.61$.
- So, the probability that for at least one planet the ratio is that close is equal to $1 - 0.61 = 0.39$.
- This is much larger that the 0.05 threshold.

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