Mathematical and Computational Aspects of a Joint Inversion Paper by M. Moorkamp, A. G. Jones, and S. Fishwick

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1. Formulation of the Geophysical Problem

- *Problem:* we are interested in some quantity q.
- *Example:* we are interested in how the density ρ depends on the depth d: $\rho = \rho(d)$.
- *Situation:* we have several types t of measurement results t, e.g., they use seismic data, resistivity, etc.
- *Measurement results:* for each type of data t, we have measurement results $m_{t,i}$, $i = 1, \ldots, n_t$.
- *Measurement accuracy:* for each measurement, we have estimates $\sigma_{t,i}$ of the accuracy of this measurement.
- *Problem:* sometimes, we only have a general accuracy estimate σ_t for all measurements of type t.
- *Solution:* in this case, we take $\sigma_{t,i} \approx \sigma_t$.

2. Formulation of the Geophysical Problem (cont-d)

- *Reminder:*
  - we are interested in a quantity q;
  - we have measurement results $m_{t,i}$ of different types t;
  - we know (approximately) the accuracies $\sigma_{t,i}$ of different measurements.
- *Forward models* M_t enables us, given q, to predict the corresponding measured values

$$m_{i,t} \approx M_t(i, q).$$

- *Least Squares formulation:* find q that minimizes

$$\sum_t \Phi_t, \text{ where } \Phi_t \stackrel{\text{def}}{=} \sum_{i=1}^{n_t} \frac{(m_{t,i} - M_t(i,q))^2}{\sigma_{t,i}^2}.$$

- *Problem:* the accuracies $\sigma_{t,i}$ are only approximately known.

3. Main Idea of the Paper

- *Ideal case:* if we knew the exact accuracies $\sigma_{t,i}$, we could apply the Least Squares approach.
- *In practice:* we only know approximate values of $\sigma_{t,i}$.
- *Reason:* for some t, we systematically overestimate the measurement errors; for other t, we underestimate.
- Whether we over- or under-estimate depends on t.
- *Natural idea:* assume that the actual accuracies are $\sigma^{\text{act}}_{t,i} = k_t \cdot \sigma_{t,i}$.
- *Resulting solution:* for all possible combinations of the correction coefficients k_t, find q that minimizes

$$\sum_t \frac{1}{k_t^2} \cdot \Phi_t, \text{ where } \Phi_t = \sum_{i=1}^{n_t} \frac{(m_{t,i} - M_t(i,q))^2}{\sigma^2_{t,i}}.$$

- *Selection* of an appropriate solution ("model") q is made by a geophysicist.

4. Pareto Optimality

- *Reminder:* for all possible combinations of the correction coefficients k_t, find q that minimizes

$$\sum_t \frac{1}{k_t^2} \cdot \Phi_t, \text{ where } \Phi_t = \sum_{i=1}^{n_t} \frac{(m_{t,i} - M_t(i,q))^2}{\sigma_{t,i}^2}.$$

- *Known:* this is $\Leftrightarrow$ finding all *Pareto optimal* solutions q, i.e., q which are not *worse* than any other q':

$$q \text{ worse than } q' \Leftrightarrow (\Phi_t(q) \le \Phi_t(q') \text{ for all } t \text{ and}$$

$$\Phi_t(q) < \Phi_t(q') \text{ for some } t).$$

- *How they find it:* use genetic algorithm, with the minimized function $O(q) \stackrel{\text{def}}{=} \#\{q' : q' \text{ worse than } q\}$.

5. Genetic Algorithm: Brief Description

- Each q is a sequence of values: e.g., $\rho(d_i)$ at different depths d_i.
- We start with several randomly generated sequences.
- At each step, we repeatedly
  - select two sequences s_1 and s_2 – the smaller $O(q)$, the larger probability of selection;
  - select random splitting locations, so $s_i = s_{i1}s_{i2}s_{i3}\ldots$, where s_{i1} is before the 1st location, etc.;
  - combine s_1 and s_2 into a new sequence $s_{11}s_{22}s_{13}s_{24}\ldots$;
  - mutate, i.e., randomly change some elements of the new sequence.
- These new sequences form a new generation, with which we deal on the next step.
- We repeat this procedure many ($N \gg 1$) times.

6. Selecting a Single Model

- *Reminder:* we find all the solutions which are Pareto-optimal with respect to $\Phi = (\Phi_1, \Phi_2, \ldots)$.
- *Interesting case:* when we have two types of measurements.
- *In this case:* we find all the solutions which are Pareto-optimal with respect to $\Phi = (\Phi_1, \Phi_2)$.
- *Empirical fact:*
  - if we plot the dependence of $\ln(\Phi_1)$ on $\ln(\Phi_2)$, then
  - at the geophysically most meaningful solution, the corresponding curve has the largest curvature.
- *Name:* the corresponding curve is called an *L-curve*, since it has a sharp corner – like a letter L.
- *Resulting idea:* look for the solution at which the curvature is the largest.