How Time Variability of Temperature Depends on a Spatial Location: A Model and Preliminary Results of Its Testing

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1. How Temperatures Change from One Spatial Location to Another: A Model

- Each environmental characteristic q changes from one spatial location to another.
- A large part of this change is unpredictable (i.e., random).
- A reasonable value to describe the random component of the difference $q(x) - q(x')$ is the variance

  $$V(x, x') \stackrel{\text{def}}{=} E[((q(x) - E[q(x)]) - (q(x') - E[q(x')]))^2].$$

- *Comment:* we assume that averages are equal.
- Locally, processes should not change much with shift $x \to x + s$: $V(x + s, x' + s) = V(x, x')$.
- For $s = -x'$, we get $V(x, x') = C(x - x')$ for

  $$C(x) \stackrel{\text{def}}{=} V(x, 0).$$

2. A Model (cont-d)

- In general, the further away the points x and x', the larger the difference $C(x - x')$.
- In the isotropic case, $C(x - x')$ depends only on the distance $D = |x - x'|^2 = (x_1 - x'_1)^2 + (x_2 - x'_2)^2$.
- It is reasonable to consider a scale-invariant dependence $C(x) = A \cdot D^\alpha$.
- In practice, we may have more changes in one direction and less change in another direction.
- E.g., 1 km in x is approximately the same change as 2 km in y.
- The change can also be mostly in some other direction, not just x- and y-directions.
- Thus, in general, in appropriate coordinates (u, v), we have $C = A \cdot D^\alpha$ for $D = (u - u')^2 + (v - v')^2$.

3. Model: Final Formulas

- In general, $C = A \cdot D^\alpha$, for $D = (u - u')^2 + (v - v')^2$ in appropriate coordinates (u, v).
- In the original coordinates x_1 and x_2, we get:

$$C(x - x') = A \cdot D^\alpha, \text{ where}$$

$$D = \sum_{i=1}^{2} \sum_{j=1}^{2} g_{ij} \cdot (x_i - x'_i) \cdot (x_j - x'_j) =$$

$$g_{11} \cdot (x_1 - x'_1)^2 + 2g_{12} \cdot (x_1 - x'_1) \cdot (x_2 - x'_2) + g_{22} \cdot (x_2 - x'_2)^2.$$

- From the computational viewpoint, we can include A into g_{ij} if we replace g_{ij} with $A^{1/\alpha} \cdot g_{ij}$, then

$$C(x - x') =$$

$$\left(g_{11} \cdot (x_1 - x'_1)^2 + 2g_{12} \cdot (x_1 - x'_1) \cdot (x_2 - x'_2) + g_{22} \cdot (x_2 - x'_2)^2\right)^\alpha$$

4. Testing the Model

- The above model describes between-station variance

$$C(x-x') = \frac{1}{T} \cdot \sum_{t=1}^{T} ((q(x,t)-\overline{q}(x)) - (q(x',t)-\overline{q}(x')))^2.$$

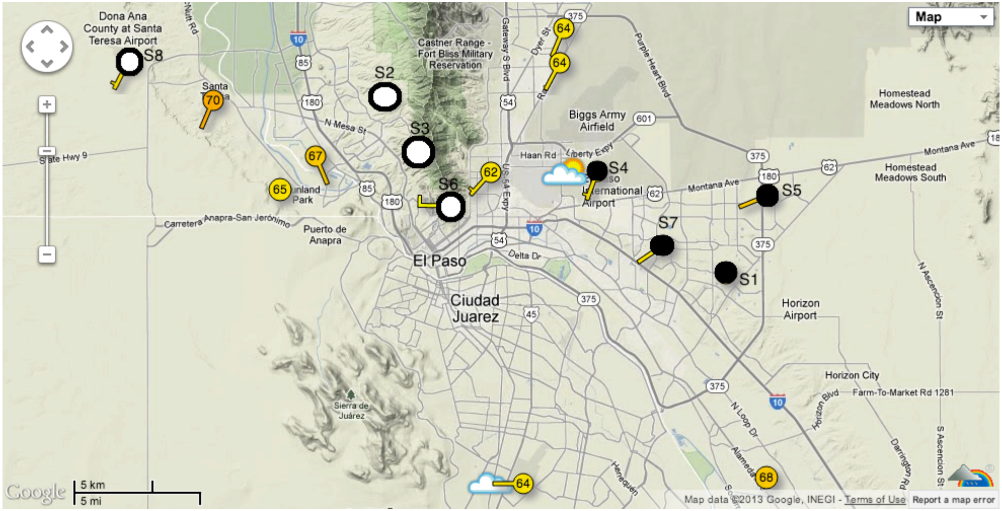
- According to the model,

$$C(x-x') \approx$$

$$\left(g_{11} \cdot (x_1 - x_1')^2 + 2g_{12} \cdot (x_1 - x_1') \cdot (x_2 - x_2') + g_{22} \cdot (x_2 - x_2')^2\right)^{\alpha}$$

- For several stations close to El Paso:
  - we estimated $C(x-x')$ and then
  - we used Least Squares to find the best fit values α and g_{ij}.
- The dependence on α is non-linear, so we tried all values $\alpha = 0.25,\ 0.3,\ 0.35,\ \ldots,\ 1.25$.

5. Current Map with 8 stations



6. Testing the Model (cont-d)

- For each α, we applied Least Squares to

$$C^{1/\alpha}(x - x') \approx$$

$$g_{11} \cdot (x_1 - x'_1)^2 + 2g_{12} \cdot (x_1 - x'_1) \cdot (x_2 - x'_2) + g_{22} \cdot (x_2 - x'_2)^2.$$

- Then, we chose α for which the resulting mean square error is the smallest.
- To check whether the model works, we compared:
  - the residual mean squared error with
  - the original mean squared value of $C(x-x')$ (which correspond to $g_{ij} \equiv 0$).
- When we considered all 8 stations, the error reduced from 5.2 to only 3.9 ($\approx 25\%$).
- When we separated the stations into E and to the W of the mountains, we got a better decrease in error.

7. Testing the Model: Results

	E	W	Both
Initial Error	2.4	6.2	5.2
Residual Error	0.23	4.3	3.9
Decrease	90%	34%	25%
α	0.80	1.00	0.35
g_{11}	2.5	0.6	12.4
g_{12}	1.7	1.7	31.0
g_{22}	1.5	9.4	88.0

8. Testing the Model: Analysis of the Results

- When we considered all eight stations, the model did not work: the error reduced from 5.2 to only 3.9.
- When we separated the stations into E and to the W of the mountains, we got a good decrease in error.
- The resulting values g_{ij} show that the dependence on spatial locations is different in two areas:
  - in E, we have $g_{11} \geq g_{12}, g_{22}$, so the main change is in E-W direction;
  - in W, we have $g_{22} \gg g_{11}, g_{12}$, so the main change is in the S-N direction.