How to Make Predictions-Related Computations Feasible: From Domain-Like Computational Structures for Modeling Space, Time, and Causality to the Use of Non-Trivial Space-Time-Causality Processes in Computations

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1. Main Problem: Introduction

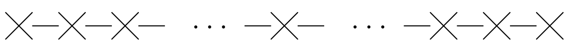
- *One of the main objectives of physics:* predict the future behavior of real-world systems.
- *Fact:* in modern physics, models for space, time, causality, and physical processes in general are very complex.
- *Example:*
  - *physical phenomenon:* a simple space-time;
  - *formalism:* quantum physics;
  - *mathematical description:* a wave function $\psi(M)$ defined on the class of all pseudo-Riemannian manifolds M.
- *Corollary:*
  - models are very complex;
  - as a result, prediction-related computations are often extremely time-consuming.
- *Difficulty:* sometimes, by the time we finish prediction computations, the predicted event has already occurred.
- *Problem:* how can we speed up these computations?

2. First Approach to Solving the Main Problem: Operationalism

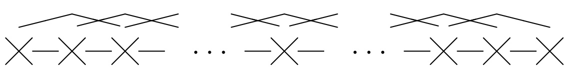
- *Fact:* in modern physics, many quantities used in the corresponding equations are not directly observable.
- *Example:* the wave function $\psi(x)$.
- *Related idea:* restrict ourselves to only computing directly observable quantities.
- *Hope:* by not computing other quantities, we can save computation time.
- *Reason for this hope:* a similar operationalistic approach – of restricting ourselves only to directly observable quantities – has been very successful in physics:
  - *special relativity:* started when Einstein analyzed observable effects of simultaneity and used relativity principle;
  - *general relativity:* started with Einstein's equivalence principle: that a person in the falling elevator does not feel any gravity;
  - *equations of quantum physics:* started with Heisenberg's matrix equations (motivated by operationalism).

3. Towards Operationalistic Approach to Computational Physics: Binary Domains

- *General idea:* in any real-life measurement, we have a finite set X of possible measurement results.
- *Description of measurement uncertainty:* $a \sim b$ if and only if the same object can lead to both measurement results a and b.
- *Physical example:*
  - *measured quantity:* temperature t°;
  - *scale:* integer values 0, 1, ..., 100;
  - *measurement accuracy:* $\pm 1^\circ$.
- *Description:* $X = \{0, 1, 2, 3, \ldots, 100\}$; $a \sim b \leftrightarrow |a - b| \le 1$.



- *Modified example:* measurement accuracy $\pm 2^\circ$.
- *Description:* $X = \{0, 1, 2, 3, \ldots, 100\}$; $a \sim b \leftrightarrow |a - b| \le 2$.



4. Towards Operationalistic Approach to Computational Physics: Binary Domains (continued)

- *Example:* counting objects up to n.
  - *Description:* $X = \{1, 2, \ldots, n-1, \text{many}\}$:

$$0 \quad 1 \qquad \cdots \qquad k \qquad \cdots \quad (n-1) \quad \text{many}$$

- *Example: "yes"-"no" questions:*
  - *possible answers:* "yes" (1), "no" (0), "unknown" (U);
  - *description:* $X = \{0, 1, U\}$, $0 \sim U \sim 1$.

$$0 \text{———} U \text{———} 1$$

- Repeated "yes"-"no" measurements: 5 possible outcomes: 0_1, 1_1, $U_1 0_2$, $U_1 1_2$, and $U_1 U_2$.
  - If the actual value is 0, we can get 0_1, $U_1 0_2$, and $U_1 U_2$;
  - if the actual value is 1, we can get 1_1, $U_1 1_2$, and $U_1 U_2$.

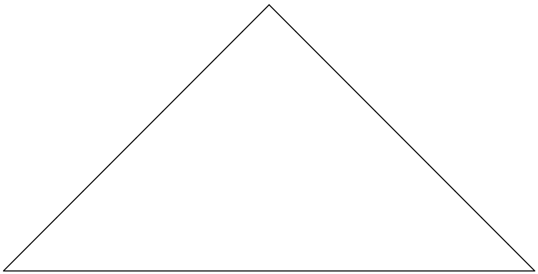
$$0_1 \text{——} U_1 0_2 \text{——} U_1 U_2 \text{——} U_1 1_2 \text{——} 1_1$$

- *General case:* graph (*web*) $\langle X, \sim \rangle$.

5. A More Adequate Description: Simplicial Complexes

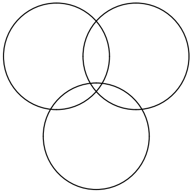
- *Previously:* we only considered compatibility of pairs of measurement results.
- *Natural idea:* consider compatibility of triples, etc.
- *Formalization:* a set $S \subseteq X$ is *compatible* if for some object, all values from S are possible after measurement.
- *Simplicial complex:* a pair $\langle X, \mathcal{S}\rangle$, where $X \subseteq \mathcal{S} \subseteq 2^X$ is the class of all compatible sets.
- *Example 1:* $X_i \cap X_j \neq \emptyset$ but $X_1 \cap X_2 \cap X_3 = \emptyset$.
- *Corresponding simplicial complex:* empty triangle $X = \{x_1, x_2, x_3\}$,

$$\mathcal{S} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}\}.$$



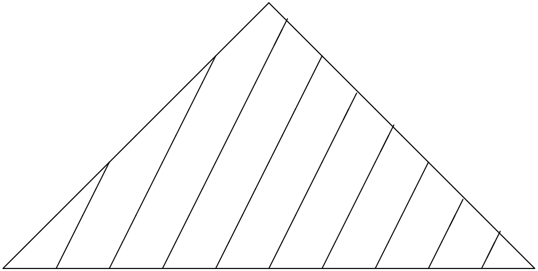
6. Simplicial Complexes (continued)

- *Example 2:* $X_1 \cap X_2 \cap X_3 \neq \emptyset$.



- *Corresponding simplicial complex:* filled triangle $X = \{x_1, x_2, x_3\}$,

$$\mathcal{S} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}, \{x_1, x_2, x_3\}\}.$$



7. How to Describe Actual Values of Measured Quantities

- *Objective:* describe actual values.
- *Problem:* single measurement leads to approximate value.
- *Solution:* consider a sequence of more and more accurate measuring instruments.
- *Relation:* let X describes results of first k measurements and X' results of $l > k$ measurements.
- The forgetful functor $\pi_{lk} : X' \to X$ is a *projection:*
  - if $a' \sim' b'$, then $\pi(a') \sim \pi(b')$;
  - if $a \sim b$, then $\exists a', b'$ s.t. $\pi(a') = a$, $\pi(b') = b$, and $a' \sim' b'$.
- *Definition:* $X_1 \stackrel{\pi_{2,1}}{\longleftarrow} X_2 \stackrel{\pi_{3,2}}{\longleftarrow} X_3 \stackrel{\pi_{4,3}}{\longleftarrow} \ldots$
- *Actual values:* $x = (x_1, x_2, \ldots)$ s.t. $\pi_{21}(x_2) = x_1, \ldots$

8. Actual Values: Properties and Examples

- *Equivalence:* $a \sim b$ iff $a_i \sim_i b_i$ for all i.
- *Neighborhoods:* $N_n(a) = \{b \,|\, b \sim_n a\}$.
- *Limit:* $a^{(k)} \to a$ iff $\forall n\, \exists m\, \forall k > m\, (a_n^{(k)} \sim_n a)$.
- *Real numbers:* naturally come from intervals.
- *Actually:* we also get $-\infty$ and $+\infty$.
- *R^n:* naturally comes from n-dimensional boxes.
- *"yes"-"no" questions:*
  - X_1: $0 \sim U \sim 1$;
  - X_2: $0 \sim U0 \sim UU \sim U1 \sim 1$, $0 \sim UU \sim 1$;
  - ...
  - projective limit: $0 \sim U \sim 1$.

9. Unusual Property: Compactness

- *General property:* every sequence has a convergent subsequence.
- *Example:* instead of R, we have a compactification $R \cup \{-\infty, +\infty\}$.
- *Potential application:* inverse problems.
- *Description:* we observe $f(x)$ for some continuous $f : X \to Y$, we want to reconstruct x.
- *Example:* signal from its distortion.
- *Problem:* even if f is 1-1, f^{-1} is discontinuous, so close y lead to different x.
- *Solution:* for compact X, f^{-1} is continuous.
- *Similar property:* $\sim$ is transitive iff

$$\forall n\, \exists m\, ((a \sim_m b\, \&\, b \sim_m c) \to (a \sim_n b).$$

10. Functions

- *Meaning:* $f : A \to B$ means that:
  - once we know an approximation a_n to a,
  - we can find some approximation b_m to b.
- *Definition:* a function $f : A \to B$ is a mapping from $\cup A_n$ to $\cup B_n$ s.t.:
  - $a \sim a'$ implies $f(a) \sim f(a')$;
  - if $a = \pi(a')$, then $f(a) = \pi(f(a'))$.
- *Comment:* functions may be partial, so the results do not converge.
- *Everywhere defined:* if $f : X \to R$ is everywhere defined, then f is continuous:

$$\forall n\, \exists m\, ((x_m \sim_m x'_m) \to f(x_m) \sim_n f(x'_m)).$$

11. First Approach: Summary

- *Idea:* restrict ourselves to directly observable results.
- *Measuring instrument:* a finite graph (or, more generally, a simplicial complex) in which:
  - vertices are possible measurement results and
  - vertices a and b are connected by an edge iff a and b can come from measuring the same quantity.
- *Physical quantity:* a sequence of more and more accurate measuring instruments.
- *Resulting mathematical representation:* a projective limits of the corresponding graphs (or complexes).
- *Resulting theory:* domain-like.
- *Computational advantage:*
  - *mathematical fact:* higher order objects like functions or operators can be described by similar graphs (complexes);
  - *computational advantage:* such higher order objects are algorithmically computable.

12. Still Too Much Computation Time

- *Problem:* computations still require a lot of computation time.
- *Especially important:* for space-time models.
- *Reason (explained below):* space-time models require more computations than spatial models.
- *Fact:* we need at least as many computation steps as points in discretized space-time.
- *Objective of discretization:* reproduce distance $d(a, b)$ with given accuracy $\varepsilon > 0$.
- *Known fact:* for a bounded area of R^n (or a Riemann space), we need $N \sim \varepsilon^{-n}$ points (e.g., a grid).
- *Space-time:* to represent proper time $\tau(a, b)$ with accuracy ε, we need $N \sim \varepsilon^{-2n}$ ($\gg \varepsilon^{-n}$) points.
- *Conclusion:* we need 'square times' more points to represent (hence, to process) space-times.
- *Reason:* e.g., in 2-D, $\tau(a, 0) = \sqrt{a_1^2 - a_2^2} = \sqrt{(a_1 + a_2) \cdot (a_1 - a_2)}$, so if $a_1 + a_2 \approx 1$ and $a_1 - a_2 \approx 0$, we need to know $a_1 - a_2$ with accuracy ε^2 to get $\tau(a, 0)$ with accuracy ε.

13. Second Approach: Predictability as a Physical Law

- *In theory:* some prediction problems are NP-hard – hence require a lot of computation time (exhaustive search).
- *In practice:* we can actually predict.
- *Idea:* treat this "predictability" as a new physical "law" (similar to anthropic principle).
- *First success:* many un-explained facts from physics (e.g., Dirac's relations between large numbers) can be thus explained.
- *First large number:* the number of time moments in the Universe $N_t = T/\Delta t \approx 10^{40}$, where:
  - $T \approx 10^{10}$ years is the lifetime of the Universe;
  - $\Delta t = \hbar/(mc^2)$ is the smallest possible time quantum.
- *Observation:* exhaustive search of n-bit strings requires $N = 2^n$ moments of time.
- *Hence:* during time N, we can process strings of size $n = \log_2(N)$.
- *Dirac's observation:* $\log_2(N) \approx 137 = 1/\alpha$, where α is the fine structure constant ($1/\alpha$ is the size of the largest possible atom).

14. Predictability as a Physical Law (continued)

- *In statistical physics:* events with very small probability $p \le p_0$ are considered impossible.
  - *mathematically:* due to Brownian motion, I can fly away;
  - *in practice:* I will not fly away.
- *How to estimate p_0?*
  - in quantum physics, locations are probabilistic;
  - in the first approximation, we have normal distributions $\rho(\vec{x}) = (\sqrt{2\pi})^{-3} \cdot \sigma^{-3} \cdot \exp(-(\vec{x} - \vec{x}_0)^2/(2\sigma^2))$;
  - thus, the smallest possible size ε_0 is when $\sigma^{-3} \cdot \varepsilon_0^3 \approx p_0$;
  - $\varepsilon_0 = \sqrt{\hbar \cdot \gamma / c^3} \approx 10^{-33}$ cm, $\sigma \approx 10^{-13}$ cm, so $p_0 \approx 10^{-60}$.
- *Alternative estimate:*
  - by Schroedinger's equation, $\Delta x(t) = \sqrt{\Delta x \cdot (c \cdot t)}$;
  - after time T_0, the probability to find a particle within its normal range becomes $\le p_0$;
  - comparing T_0 to $T = 10^{10}$ years, we also get $p_0 \approx 10^{-60}$.
- *Conclusion:* the equality of two estimates for p_0 explains another Dirac's relation.

15. Third Approach: Using Under-Utilized Space-Time-Causality Processes for Computation

- *Problem:* even with the restrictions on physical models, computations may take a long time.
- *Explanation:* traditional computational complexity estimates are based on traditional physics and traditional space-time.
- *Known fact:* quantum effects can drastically speed up computations:
  - time needed for search in an unsorted list of size n is reduced from n to $\sqrt{n}$;
  - time needed to factor large large numbers is reduced from exponential to polynomial, etc.
- *Less known fact:* space-time-causality processes can also lead to a drastic computational speed up.
- *Examples:*
  - highly curved space-times;
  - acausal processes.

16. Parallelization: Reminder

- *Good news:*
  - parallel computers can speed up computations;
  - the more processors, the faster computations.
- *Seemingly natural hypothesis:* if we accumulate a lot of processors, we solve exponential-time problems in polynomial time.
- *Result:* parallelism cannot reduce the computation time T that drastically.
- *Analysis:*
  - during the parallel computation time T_p, we can only access computers within a sphere of radius $R = c \cdot T_p$;
  - within this sphere of volume $V = \dfrac{4}{3} \cdot \pi \cdot R^3 \sim T_p^3$, we can only fit $\le V/\Delta V \sim T_p^3$ processors of given size ΔV;
  - all these processors can perform $T \le T_p \cdot \text{const} \cdot T_p^3 = C \cdot T_p^4$ computational steps.
- *Conclusion:* if a computation requires T sequential steps, we need $T_p \ge C \cdot T^{1/4}$ steps to perform it in parallel.

17. Parallelization in Curved Space-Times

- *Observation:* the above lower bound on parallel computation time depends on the formula $V(R)=\frac{4}{3}\cdot\pi\cdot R^3$.
- *Known:*
  - this formula only holds in Euclidean geometry, and
  - actual space-time is curved (= not Euclidean).
- *Natural idea:* we may get faster parallel computations in curved spaces.
- *Known:* in Lobachevsky space,

$$V(R)=2\pi k^3\cdot\left(\sinh\left(\frac{R}{k}\right)\cdot\cosh\left(\frac{R}{k}\right)-\frac{R}{k}\right)\sim\exp\left(\frac{2}{k}\cdot R\right).$$

- *Corollary:* we can fit exponentially many processors into a sphere of radius $R=c\cdot T_p$.
- *Conclusion:* in Lobachevsky space, parallelization can reduce exponential time $T=2^n$ to linear time $T_p\sim n$.
- *Lobachevsky's observation:* by measuring $V(R)$, we can speed up computation of hyperbolic trigonometric functions.

18. Parallelization in Curved Space-Times (continued)

- *Good news:* in Lobachevsky (constant curvature) space, parallelization speeds up computations.
- *Problem:* actual space-time is more complex.
- *Good news:* there exist more realistic space-time models with the same property.
- *Known:* there is no way to escape from a black hole.
- *Known:* as the matter collapses, the escape throat gets narrower.
- *Less known:* there exist "almost" black hole models, with a throat so narrow that they look like elementary particles.
- *Known hypothesis:* particles are such "almost" black holes, entering into other "universes".
- To find $x = (x_1, \ldots, x_n)$, $x_i \in \{0, 1\}$, s.t. $F(x)$, we:
  - find two particles (and corr. worlds);
  - ask World 1 to search for $x = (0, x_2, \ldots, x_n)$ s.t. $F(x)$;
  - ask World 2 to search for $x = (1, x_2, \ldots, x_n)$ s.t. $F(x)$.
- Each of these worlds does the same split w.r.t. x_2, etc.; in time $2n$ ($\ll 2^n$), we get an answer back.

19. Another Idea: Using Hypothetic Acausal Processes

- *Known fact:* several physical theories have led to micro- and macro-causality violations, i.e., going back in time.
- *Feynman:* positrons are electrons going back in time.
- *Mainstreaming:* K. Thorne's Physical Reviews papers.
- *General relativity:* space-time generated by a massive fast-rotating cylinder contains a closed timelike curve.
- *String theory*: interactions between string-like particles sometime lead to the possibility to influence the past.
- *Cosmology:* a short period of exponentially fast growth ("inflation") preceding current can lead to a causal anomaly.
- *Paradox of causality violation:* a time traveler goes into the past and kills his father before he himself was conceived.
- *Solution:* since the time traveler was born, some unexpected event prevented him from killing his father.
- The time traveler takes care of all such probable events.
- *But:* there are always events with small probability – a meteor falling on the traveler's head – which cannot all be avoided.

20. Using Acausal Processes for Computations

- *Example:* propositional satisfiability problem with n variables.
- *Straightforward option:* computer computes and send result back in time, to us now.
- *Problem:* with time travel, we may invoke an event with a very small probability $p_0 \ll 1$, and ruin computations.
- *Alternative algorithm:*
  - generate n random bits $x_1, \ldots, x_n$ and check whether they satisfy a given formula F;
  - if not, launch a time machine that is set up to implement a low-probability event.
- *Analysis:* nature has two choices:
  - generates n variables which satisfy the given formula (probability 2^{-n}),
  - time machine is used, triggering an event with probability p_0.
- If $2^{-n} \gg p_0$, then the first event is much more probable.
- So, the solution to the satisfiability problem will actually be generated (without the actual use of a time machine).

21. Acknowledgments

This work was supported in part:

- by National Science Foundation grants EAR-0225670 and DMS-0532645 and
- by Texas Department of Transportation grant No. 0-5453

22. Further Reading

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