Standing on the Shoulders of the Giants: From Einstein's General Relativity and Zadeh's Fuzzy Logic to Computers of Generation Omega

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Albert Einstein and . . .

Their Ideas Are Not As . . .

Einstein – Zadeh – . . .

The Main Challenge . . .

Is Parallelization a . . .

Einstein Can Help: . . .

Acausal Processes: . . .

No Physical Theory Is . . .

Conclusion: . . .

1. Which Problems Are Most Significant?

- In 1986, Richard Hamming, of the Hamming code fame, gave a talk titled You and Your Research.

- In this talk, he emphasized that to become a great scientist, it is important to work on significant problems.

- So which problems are most significant?

- We want to know how the world functions, what is the causal relation between different processes.

- In analyzing causality, the most revolutionary results were obtained by Einstein.

- We also want to know how we humans functions, how we reason, how we make decisions.

- In describing human reasoning, probably the most revolutionary idea is Zadeh's idea of fuzzy logic.

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2. Albert Einstein and Lotfi Zadeh

- At present, we celebrate 100 years of general relativity and 50 years of fuzzy.

- It is thus time to compare their authors.

- At first glance, their ideas was diametrically opposite:

  - Einstein was a known enemy of uncertainty, he even objected to quantum physics, while

  - Zadeh emphasized uncertainty.

- But on a deeper level, their ideas are similar: they both emphasize the need to challenge the prevailing dogmas.

- Einstein showed that the physical world is not described by Euclidean geometry.

- Zadeh showed that our reasoning is not described by 2-valued logic.

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3. Their Ideas Are Not As Radical As They May Seem

- Interestingly, it later turned out that their theories are not as revolutionary as one might think.

- Einstein's equations can be deduced from the field theory if we assume that energy is the field's source.

- Zadeh's logic can be described in traditional terms – and Lukaciewicz has done that already in 1920s.

- So why are they famous? Why are their ideas widely used?

- Because both were motivated by *applications.*

- Hilbert discovered Einstein's equations 2 weeks after Einstein – but he did not have physical applications.

- Lukaciewicz proposed fuzzy logic 40 years before Zadeh – but he did not have applications.

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4.   Einstein – Zadeh – What Next?

- What are significant problems now?

- In Einstein's time, the main challenge was to come up with equations that describe the physical world.

- After relativity and quantum physics, we have pretty accurate equations.

- The main challenge now is how to *use* these equations, how to predict events using these equations.

- In principle, we know the equations describing weather.

- However, modern supercomputers barely have time predict tomorrow's weather.

- In principle, we can predict whether a tornado will turn in the next 15 minutes.

- However, on modern supercomputers, these computations require several days.

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5. The Main Challenge Facing Science: How to Compute Faster

- So, the main challenge now is how to compute faster.

- Theoretical computer science's study of NP-hardness has shown that many problems are inherently complex.

- This means that we cannot decrease the number of computational steps.

- So, the only way to compute faster is to design faster computers.

- How is this done now? The first natural idea is to have several processors working in parallel.

- This is why our brain can solve some problems faster than a supercomputer.

- Another idea is miniaturization: a 30 cm laptop requires 1 nanosecond for the signal to pass through.

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6.   Is Parallelization a Panacea?

- If we accumulate a lot of parallel processors, maybe we solve exponential-time problems in polynomial time?

- *Result:* parallelism cannot reduce the computation time T that drastically.

- During the parallel computation time T_p, we can only access computers within a sphere of radius $R = c \cdot T_p$.

- Within this sphere of volume $V = \dfrac{4}{3} \cdot \pi \cdot R^3 \sim T_p^3$, we can fit $\leq V/\Delta V \sim T_p^3$ processors of given size ΔV.

- All these processors can perform $T \leq \dfrac{T_p}{\Delta t} \cdot \text{const} \cdot T_p^3 = C \cdot T_p^4$ computational steps.

- So, if a computation requires T sequential steps, we need $T_p \geq C \cdot T^{1/4}$ steps to perform it in parallel.

7. Einstein Can Help: Curved Space-Time

- *Observation:* the above lower bound on parallel computation time depends on the formula $V(R) = \frac{4}{3} \cdot \pi \cdot R^3$.

- *Known:* this formula only holds in Euclidean geometry.

- *Idea:* since the actual space-time is curved (= not Euclidean), we may get faster parallel computations.

- *Known:* in Lobachevsky space,

$$V(R) = 2\pi k^3 \cdot \left(\sinh\left(\frac{R}{k}\right) \cdot \cosh\left(\frac{R}{k}\right) - \frac{R}{k} \right) \sim \exp\left(\frac{2}{k} \cdot R\right).$$

- *Corollary:* we can fit exponentially many processors into a sphere of radius $R = c \cdot T_p$.

- *Conclusion:* in Lobachevsky space, parallelization can reduce exponential time $T = 2^n$ to linear time $T_p \sim n$.

- *Lobachevsky's idea:* by measuring $V(R)$, we can speed up computation of $\sinh(x)$.

8.   Parallelization in Curved Space-Time (cont-d)

- *Assumption:* particles are such "almost" black holes, entering into other "universes".

- Let us show how this can help solve NP-hard problems, on the example of propositional satisfiability SAT:

    – given a propositional formula $F(x_1, \ldots, x_n)$,

    – find the values $x_1, \ldots, x_n$ s.t. $F(x)$ is true.

- To find $x = (x_1, \ldots, x_n)$, $x_i \in \{0, 1\}$, s.t. $F(x)$, we:

    – find two particles (and corr. worlds);

    – ask World 1 to search for $x = (0, x_2, \ldots, x_n)$ s.t. $F(x)$;

    – ask World 2 to search for $x = (1, x_2, \ldots, x_n)$ s.t. $F(x)$.

- Each of these worlds does the same split w.r.t. x_2, etc.; in time $2n$ ($\ll 2^n$), we get an answer back.

9. Acausal Processes: Reminder

- Several physical theories have led to micro- and macro-causality violations, i.e., going back in time.

- *Feynman:* positrons are electrons going back in time.

- *Mainstreaming:* K. Thorne's Physical Reviews papers.

- *General relativity:* space-time generated by a massive fast-rotating cylinder contains a closed timelike curve.

- *String theory*: interactions between string-like particles sometime lead to the possibility to influence the past.

- *Cosmology:*
  - a short initial period of exponentially fast growth ("inflation")
  - can lead to a causal anomaly.

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10.   Acausal Processes: Analysis

- *Paradox of causality violation:*
    - a time traveler goes into the past and
    - kills his father before he himself was conceived.

- *Solution:* since the time traveler was born, some unexpected event prevented him from killing his father.

- The time traveler takes care of all such probable events.

- *But:* we cannot avoid all events with small probability.

- *Example:* a meteor can fall on the traveler's head and prevent him from killing his father.

- *Conclusion:* time travel may be possible.

- *How to use it for computations:* a computer computes and send the result back in time, to us now.

11.　Using Acausal Processes for Computations

- *Alternative algorithm* for solving SAT:
  - generate n random bits $x_1, \ldots, x_n$ and check whether they satisfy a given formula $F(x_1, \ldots, x_n)$;
  - if not, launch a time machine that is set up to implement a low-probability event.

- *Analysis:* nature has two choices:
  - generates n variables which satisfy the given formula (probability 2^{-n}),
  - time machine is used, triggering an event with probability p_0.

- If $2^{-n} \gg p_0$, then the first event is much more probable.

- So, the solution to the satisfiability problem will actually be generated.

- *Interesting:* there is no actual time travel.

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12. No Physical Theory Is Perfect

- If a speed-up is possible within a given theory, is this a satisfactory answer?

- In the history of physics,

    - always new observations appear

    - which are not fully consistent with the original theory.

- For example, Newton's physics was replaced by quantum and relativistic theories.

- Many physicists believe that every physical theory is approximate.

- For each theory T, inevitably new observations will surface which require a modification of T.

- This Zadeh-type idea helps compute faster!

13.   Conclusion: Computers of Generation Omega

- One of the main challenges of modern science is the need to compute faster.

- We have gone through several generations of computers: tubes, semiconductors, chips, etc.

- Engineers are working on the next generation.

- We scientists should also think of the distant future: computers of generation omega.

- The main point of this talk is that these computers will be based on the legacy of Einstein and Zadeh:

  – they will use curved space-time, black holes, and causality violations related to Einstein's GRT,
  – they will also use Zadeh's idea: no theory is perfect.

- This will confirm what David Hilbert said in his famous 1900 address: There is no ignorabimus! We will know!

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16. Appendix: Formalizing the Idea that No Physical Theory Is Perfect

- *Statement:* for every theory, eventually there will be observations which violate this theory.

- To formalize this statement, we need to formalize what are *observations* and what is a *theory*.

- Most sensors already produce *observation* in the computer-readable form, as a sequence of 0s and 1s.

- Let ω_i be the bit result of an experiment whose description is i.

- Thus, all past and future observations form a (potentially) infinite sequence $\omega = \omega_1 \omega_2 \ldots$ of 0s and 1s.

- A physical *theory* may be very complex.

- All we care about is which sequences of observations ω are consistent with this theory and which are not.

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17.   What Is a Physical Theory?

- So, a physical theory T can be defined as the set of all sequences ω which are consistent with this theory.

- A physical theory must have at least one possible sequence of observations: $T \neq \emptyset$.

- A theory must be described by a finite sequence of symbols: the set T must be *definable*.

- How can we check that an infinite sequence $\omega = \omega_1 \omega_2 \ldots$ is consistent with the theory?

- The only way is check that for every n, the sequence $\omega_1 \ldots \omega_n$ is consistent with T; so:

$$\forall n \, \exists \omega^{(n)} \in T \, (\omega_1^{(n)} \ldots \omega_n^{(n)} = \omega_1 \ldots \omega_n) \Rightarrow \omega \in T.$$

- In mathematical terms, this means that T is *closed* in the Baire metric $d(\omega, \omega') \stackrel{\text{def}}{=} 2^{-N(\omega, \omega')}$, where

$$N(\omega, \omega') \stackrel{\text{def}}{=} \max\{k : \omega_1 \ldots \omega_k = \omega_1' \ldots \omega_k'\}.$$

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18. What Is a Physical Theory: Definition

- A theory must predict something new.

- So, for every sequence $\omega_1 \ldots \omega_n$ consistent with T, there is a continuation which does not belong to T.

- In mathematical terms, T is *nowhere dense.*

- *By a* physical theory, *we mean a non-empty closed nowhere dense definable set T.*

- *A sequence ω is* consistent with the no-perfect-theory principle *if it does not belong to any physical theory.*

- In precise terms, ω does not belong to the union of all definable closed nowhere dense set.

- There are countably many definable set, so this union is *meager* (= *Baire first category*).

- Thus, due to Baire Theorem, such sequences ω exist.

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19. How to Represent Instances of an NP-Complete Problem

- For each NP-complete problem $\mathcal{P}$, its instances are sequences of symbols.

- In the computer, each such sequence is represented as a sequence of 0s and 1s.

- We can append 1 in front and interpret this sequence as a binary code of a natural number i.

- In principle, not all natural numbers i correspond to instances of a problem $\mathcal{P}$.

- We will denote the set of all natural numbers which correspond to such instances by $S_{\mathcal{P}}$.

- For each $i \in S_{\mathcal{P}}$, we denote the correct answer (true or false) to the i-th instance of the problem $\mathcal{P}$ by $s_{\mathcal{P},i}$.

20. What We Mean by Using Physical Observations in Computations

- In addition to performing computations, our computational device can:

  – produce a scheme i for an experiment, and then
  – use the result ω_i of this experiment in future computations.

- In other words, given an integer i, we can produce ω_i.

- In precise terms, the use of physical observations in computations means that use ω as an *oracle*.

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21. Result

- A *ph-algorithm* $\mathcal{A}$ is an algorithm that uses an oracle ω consistent with the no-perfect-theory principle.

- The result of applying an algorithm $\mathcal{A}$ using ω to an input i will be denoted by $\mathcal{A}(\omega, i)$.

- We say that a feasible ph-algorithm $\mathcal{A}$ *solves almost all instances of an NP-complete problem* $\mathcal{P}$ if:

$$\forall \varepsilon_{>0} \, \forall n \, \exists N_{\geq n} \, \left(\frac{\#\{i \leq N : i \in S_{\mathcal{P}} \, \& \, \mathcal{A}(\omega, i) = s_{\mathcal{P}, i}\}}{\#\{i \leq N : i \in S_{\mathcal{P}}\}} > 1 - \varepsilon \right).$$

- Restriction to sufficiently long inputs $N \geq n$ makes sense: for short inputs, we can do exhaustive search.

- **Theorem.** *For every NP-complete problem $\mathcal{P}$, there is a feasible ph-alg. $\mathcal{A}$ solving almost all instances of $\mathcal{P}$.*