

How to Explain the Anchoring Formula in Behavioral Economics

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1. What Is Anchoring Effect?

- Traditional economics assumes:
 - that people know the exact value of each possible item, and
 - that this value determines the price that they are willing to pay for this item.
- The reality is more complicated.
- In many practical situations:
 - people are uncertain about the value of an item, and thus,
 - uncertain about the price they are willing to pay for this item.
- This happens, e.g., when hunting for a house.

2. What Is Anchoring Effect (cont-d)

- In many such situations, the price that the customer is willing to pay is affected by the asking price:
 - if the asking price is higher, the customer is willing to pay a higher price, but
 - if the asking price is lower, the price that the customer is willing to pay is also lower.
- This phenomenon is known as the *anchoring effect*:
 - just like a stationary ship may move a little bit, but cannot move too far away from its anchor,
 - similarly, a customer stays closer to the asking price – which thus acts as a kind of an anchor.
- The anchoring effect may sound somewhat irrational.
- However, it makes some sense.

3. What Is Anchoring Effect (cont-d)

- If the owner lists his/her house at an unexpectedly high price, then maybe:
 - there are some positive features of the house
 - of which the customer is not aware.
- After all, the owner does want to sell his/her house.
- So he/she would not just list an outrageously high price without any reason.

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4. What Is Anchoring Effect (cont-d)

- Similarly, if the owner lists his/her house at an unexpectedly low price, then maybe:
 - there are some drawbacks of the house or of its location
 - of which the customer is not aware.
- After all, the owner does want to get his/her money back when selling his/her house.
- So he/she would not just list an outrageously low price without any reason.

5. A Formula Describing the Anchoring Effect

- Let p_0 be the price that the customer would suggest in the absence of an anchor.
- Of course:
 - if the asking price a is the same value $a = p_0$,
 - there is no reason for the customer to change the price p that he/she is willing to pay for this item,
 - i.e., this price should still be equal to p_0 .
- It turns out that each anchoring situation can be described by a coefficient $\alpha \in [0, 1]$.
- This coefficient is called an *anchoring index*.
- The idea is that:
 - if we consider two different asking prices $a' \neq a''$,
 - then the difference $p' - p''$ between the resulting customer's prices should be equal to $\alpha \cdot (a' - a'')$.

6. Anchoring Effect Formula (cont-d)

- We know that $p = p_0$ when $a = p_0$.
- Thus, the above idea enables us to come up with the formula describing the anchoring effect.
- Indeed, for anchor a , the difference $p - p_0$ is equal to

$$\alpha \cdot (a - p_0).$$

- Since $p - p_0 = \alpha \cdot (a - p_0)$, we thus have

$$p = p_0 + \alpha \cdot (a - p_0) = (1 - \alpha) \cdot p_0 + \alpha \cdot a.$$

- How can we explain this empirical formula?

7. What Are the Values of the Anchoring Index

- In different situations, we observe different values of the anchoring index.
- When people are not sure about their original opinion, the anchoring index is usually close to 0.5.
- For a regular person buying a house, this index is equal to $0.48 \approx 0.5$.
- For people living in a polluted city,
 - when asked what living costs they would accept to move to an environmentally clean area,
 - the anchoring index was also close to 0.5.

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8. Values of the Anchoring Index (cont-d)

- For other situations:
 - when a decision maker is more confident in his/her original opinion,
 - we can get indices between 0.25 and 0.5.
- For a real estate agent buying a house, this index is equal to 0.41.
- For a somewhat similar situation of charity donations, this index is equal to 0.30.
- How can we explain these values?

9. What We Do in This Talk

- In this talk, we try our best to answer both questions.
- We provide a formal explanation for the general formula.
- We provide a somewhat less formal explanation for the empirically observed values of the anchoring index.
- To make our explanations more convincing, we have tried to make the mathematics as simple as possible.

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10. What We Want

- We are given two numbers:
 - the price p_0 that the customer is willing to pay before getting the asking price, and
 - the actual asking price a ,
- We want a function that, given p_0 and a , produces the price $p(p_0, a)$ that the customer is willing to pay.

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11. Three Natural Properties

- As we have mentioned, if $a = p_0$, then we should have

$$p(p_0, a) = p(p_0, p_0) = p_0.$$

- Small changes in p_0 and a should not lead to drastic changes in the resulting price.
- In mathematical terms, this means that the function $p(p_0, a)$ should be continuous.
- Intuitively, the change from p_0 to p should be in the direction to the anchor, i.e.:
 - if $a < p_0$, we should have $p(p_0, a) \leq p_0$, and
 - if $p_0 < a$, we should have $p_0 \leq p(p_0, a)$.
- When the changed value $p(p_0, a)$ moves in the direction of the asking price a , it should not exceed a :
 - if $a < p_0$, we should have $a \leq p(p_0, a)$, and
 - if $p_0 < a$, we should have $p(p_0, a) \leq a$.

12. Three Natural Properties (cont-d)

- These three property can be summarized by saying that:
 - for all p_0 and a ,
 - the price $p(p_0, a)$ should always be in between the original price p_0 and the asking price a .

13. Fourth Natural Property: Additivity

- Suppose that we have two different situations.
- For example. a customer is buying two houses, a house to live in and a smaller country house for vacationing.
- Suppose that:
 - for the first item, the original price was p'_0 and the asking price is a' , and
 - for the second item, the original price was p''_0 and the asking price is a'' .
- The price of the first item is $p(p'_0, a')$, the price of the second item is $p(p''_0, a'')$, thus the overall price is:

$$p(p'_0, a') + p(p''_0, a'').$$

- Alternatively, instead of considering the two items separately, we can view them as a single combined item.

14. Additivity (cont-d)

- This combined item has the original price $p'_0 + p''_0$ and the asking price $a' + a''$.
- From this viewpoint, the resulting overall price of both items is $p(p'_0 + p''_0, a' + a'')$.
- Since these two prices correspond to the exact same situation, it is reasonable to require that they coincide:

$$p(p'_0, a') + p(p''_0, a'') = p(p'_0 + p''_0, a' + a'').$$

- Now, we are ready to formulate and prove our main result.

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15. Definitions and the Main Result

- A continuous function $p : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ is called an anchoring function if:
 - for all p_0 and a , the value $p(p_0, a)$ should always be in between p_0 and a , and
 - for all possible values $p'_0, p''_0, a',$ and a'' , we should have $p(p'_0, a') + p(p''_0, a'') = p(p'_0 + p''_0, a' + a'')$.
- **Proposition.** A function $p(p_0, a)$ is an anchoring function if and only if it has the form

$$p(p_0, a) = (1 - \alpha) \cdot p_0 + \alpha \cdot a \text{ for some } \alpha \in [0, 1].$$
- This proposition justifies the empirical expression for the anchoring effect.

16. Proof

- It is easy to see that $p(p_0, a) = (1 - \alpha) \cdot p_0 + \alpha \cdot a$ satisfies the definition and is, thus, an anchoring function.
- So, to complete the proof, it is sufficient to prove that every anchoring function has the desired form.
- Indeed, let us assume that the function $p(p_0, a)$ satisfies both conditions.
- Then, due to additivity, for each p_0 and a , we have $p(p_0, a) = p(p_0, 0) + p(0, a)$.
- Thus, to find the desired function of two variables, it is sufficient to consider two functions of one variable:

$$p_1(p_0) \stackrel{\text{def}}{=} p(p_0, 0) \text{ and } p_2(a) \stackrel{\text{def}}{=} p(0, a).$$

- Due the same additivity property, each of these functions is itself additive: $p(p'_0 + p''_0, 0) = p(p'_0, 0) + p(p''_0, 0)$,

$$p(0, a' + a'') = p(0, a') + p(0, a'').$$

17. Proof (cont-d)

- In other words, both functions $p_1(x)$ and $p_2(x)$ are additive in the sense that for each of them, we have:

$$p_i(x' + x'') = p_i(x') + p_i(x'').$$

- Since the function $p(p_0, a)$ is continuous, both functions $p_i(x)$ are continuous as well.
- Let us show that every continuous additive function is linear, i.e., has the form $p_i(x) = c_i \cdot x$ for some c_i .
- Indeed, let us denote $c_i \stackrel{\text{def}}{=} p_i(1)$.
- We have $\frac{1}{n} + \dots + \frac{1}{n}$ (n times) = 1.
- So, due to additivity:

$$p_i\left(\frac{1}{n}\right) + \dots + p_i\left(\frac{1}{n}\right) \text{ (} n \text{ times)} = p_i(1) = c_i.$$

18. Proof (cont-d)

- So, $n \cdot p_i \left(\frac{1}{n} \right) = c_i$ and thus, $p_i \left(\frac{1}{n} \right) = c_i \cdot \frac{1}{n}$.

- Similar, for every m and n , we have

$$\frac{1}{n} + \dots + \frac{1}{n} \text{ (} m \text{ times)} = \frac{m}{n}.$$

- Thus, due to additivity: we have

$$p_i \left(\frac{1}{n} \right) + \dots + p_i \left(\frac{1}{n} \right) \text{ (} m \text{ times)} = p_i \left(\frac{m}{n} \right).$$

- The left-hand side of this formula is equal to

$$m \cdot p_i \left(\frac{1}{n} \right) = m \cdot \left(c_i \cdot \frac{1}{n} \right) = c_i \cdot \frac{m}{n}.$$

- Thus, for every m and n , we have $p_i \left(\frac{m}{n} \right) = c_i \cdot \frac{m}{n}$.

- The property $p_i(x) = c_i \cdot x$ therefore holds for every rational number.

19. Proof (cont-d)

- Each real number x can be viewed as a limit of its rational approximations x_n ($x = \lim x_n$).
- Since $p_i(x)$ is continuous, in the limit, that $p_i(x) = c_i \cdot x$ for all non-negative numbers x .
- Thus, $p(p_0, 0) = p_1(p_0) = c_1 \cdot p_0$, $p(0, a) = p_2(a) = c_2 \cdot a$, and $p(p_0, a) = c_1 \cdot p_0 + c_2 \cdot a$.

- For $p_0 = a$, the requirement that $p(p_0, a)$ is between p_0 and a implies that $p(p_0, a) = p_0$, so $c_1 \cdot p_0 + c_2 \cdot p_0 = p_0$,

$$c_1 + c_2 = 1, \text{ and } c_1 = 1 - c_2.$$

- So, we get the desired formula with $c_2 = \alpha$.

20. Proof (cont-d)

- To complete the proof, we need to show that $0 \leq \alpha \leq 1$.
- Indeed, for $p_0 = 0$ and $a = 1$, the value $p(0, 1)$ must be between 0 and 1.
- Due to our formula, this value is equal to

$$(1 - c_2) \cdot 0 + c_2 \cdot 1 = c_2.$$

- Thus, $c_2 \in [0, 1]$. The proposition is proven.

21. Explaining the Numerical Values of the Anchoring Index

- Let us first consider the case when the decision maker is not sure which is more important:
 - his/her a priori guess – as reflected by the original value p_0 ,
 - or the additional information as described by the asking price a .
- In this case, in principle, the value α can take any value from the interval $[0, 1]$.
- To make a decision, we need to select one value α_0 from this interval.
- Let us consider the discrete approximation with accuracy $\frac{1}{N}$ for some large N .

22. Explaining the Numerical Values (cont-d)

- In this approximation, we only need to consider values $0, \frac{1}{N}, \frac{2}{N}, \dots, \frac{N-1}{N}, 1$, for some large N .

- If we list all possible values, we get a tuple

$$\left(0, \frac{1}{N}, \frac{2}{N}, \dots, \frac{N-1}{N}, 1\right).$$

- We want to select a single value α_0 , i.e., in other words, we want to replace the original tuple with $(\alpha_0, \dots, \alpha_0)$.
- It is reasonable to select the value α_0 for which the replacing tuple is the closest to the original tuple, i.e., for which the distance is the smallest:

$$\sqrt{(\alpha_0 - 0)^2 + \left(\alpha_0 - \frac{1}{N}\right)^2 + \dots + (\alpha_0 - 1)^2}.$$

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23. Explaining the Numerical Values (cont-d)

- Minimizing the distance is equivalent to minimizing its square

$$(\alpha_0 - 0)^2 + \left(\alpha_0 - \frac{1}{N}\right)^2 + \dots + (\alpha_0 - 1)^2.$$

- Differentiating this expression with respect to α_0 and equating the derivative to 0, we conclude that

$$2(\alpha_0 - 0) + 2\left(\alpha_0 - \frac{1}{N}\right) + \dots + 2(\alpha_0 - 1) = 0.$$

- If we divide both sides by 2 and move the terms not containing α_0 to the right-hand side, we conclude that

$$(N + 1) \cdot \alpha_0 = 0 + \frac{1}{N} + \frac{2}{N} + \dots + \frac{N - 1}{N} + 1.$$

- So, $(N + 1) \cdot \alpha_0 = \frac{1 + 2 + \dots + (N - 1) + N}{N}$.

24. Explaining the Numerical Values (cont-d)

- Thus $\alpha_0 = \frac{1 + 2 + \dots + (N - 1) + N}{N \cdot (N + 1)}$.
- It is known that $1 + 2 + \dots + N = \frac{N \cdot (N + 1)}{2}$, thus
$$\alpha_0 = 0.5.$$
- This is exactly the value used when the decision maker is not confident in his/her original estimate.

25. Second Case

- What if the decision maker has more confidence in his/her original estimate than in the anchor?
- The weight $1 - \alpha$ corr. to the original estimate must be larger than the weight α corr. to the anchor.
- The inequality $1 - \alpha > \alpha$ means that $\alpha < 0.5$.
- Similarly to the above case:
 - we can consider all possible values between 0 and 0.5, and
 - select a single value α_0 which is, on average, the closest to all these values.
- Similar to above calculations, we can conclude that the best value is $\alpha = 0.25$.

26. Second Case (cont-d)

- Correspondingly:
 - intermediate cases when the decision maker's confidence in his original opinion is somewhat larger,
 - can be described by values α between the two above values 0.5 and 0.25.
- This explains why these intermediate values occur in such situations.

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