

Chaos: what is it? is it good or bad? how to detect it? how to process data and make decisions in the presence of chaos?

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1. Science before chaos

- One of the main objectives of science and engineering is:
 - to predict the future state of the world, and
 - to make sure that this future state is maximally beneficial for humanity.
- At the beginning of science, prediction meant deterministic prediction:
 - based on the current state of the world,
 - we come up with exact predictions.
- This is how ancient researchers predicted Solar eclipses and future locations of the planets.
- This is how Newton's physics enables us to predict the trajectory of a spaceship.
- Usually, the future dynamics is described by differential equations.

2. Science before chaos (cont-d)

- In this case, predictions are made by solving these equations.
- It later turned out that for some processes, we can make only probabilistic predictions.
- One of the first such examples is radioactivity.
- It is not possible to predict when exactly a radioactive atom will go through the corresponding transformation.
- However, we can predict the probability that it will decay by a certain moment of time.
- Probabilistic predictions are ubiquitous in quantum physics.

3. Science before chaos (cont-d)

- In quantum physics, we also have differential equations that describe how the state of the world changes with time.
- The only difference is that in quantum physics:
 - even if we have complete knowledge of the current state of the world,
 - we cannot predict the exact measurement results,
 - we can only predict the probability of different measurement results.

4. Enter chaos

- Until 1970s, researchers believed that we can always predict such probabilities.
- This all changed with the famous pioneer 1972 paper by Edward Lorenz.
- This paper showed that in weather dynamics, long-terms predictions cannot be made exact.
- To make exact predictions about future weather, we need to know the initial values with impossible accuracy of plus minus 1 atom.
- A small atom-size change in the initial condition can cause a drastic change in the future weather.
- Lorenz called this phenomenon *chaos*.
- Since 1972, the chaos phenomenon was observed in many areas, including economics.
- There are many studies of chaos phenomena.

5. What we do in this talk

- In this talk, we focus on practical aspects of chaos.
- Namely, we analyze:
 - what can we predict in such situations,
 - how can we make decisions under chaos, and
 - how can we best perform the corresponding computations.

6. Is chaos good or bad?

- From the practical viewpoint, why do we need to study chaos?
- To answer this question, let us recall what is a practical need for studying different phenomena in the first place:
- Some phenomena we study because of their potential benefits.
- This is, e.g., why geoscientists look for mineral deposits: to help deliver metals and energy.
- Some phenomena we study because of their potential negative effects.
- This is, e.g., why geoscientists study earthquakes – to make sure that:
 - in each seismic area,
 - all the buildings can withstand the strongest earthquakes possible in this area.

7. Is chaos good or bad (cont-d)

- From this viewpoint, it is important to understand whether chaos is good or bad for us.
- Interestingly, the answer to this question depends on the degree of chaos.

8. Too much chaos is bad

- As we have mentioned, chaos means that:
 - instead of a single future value of a physical quantity (or probability),
 - we have a whole range of values.
- Too much chaos means that this range is very wide – i.e., in effect, that we cannot make any meaningful prediction.
- If we cannot predict what will happen, then it is difficult to make good decisions.
- For example, it has been shown that:
 - when the population of a species grows too fast,
 - we cannot really predict consequences of each action,
 - so inevitably, evolution of such a species makes a wrong decision and,
 - as a result, the species becomes extinct.

9. Too much chaos is bad (cont-d)

- Similar negative consequences appear when a country's economy starts growing too fast.
- From this viewpoint, it is desirable:
 - to avoid too much chaos, and
 - to study situations when chaos is limited.
- An example of such situation is transition to democracy: it was shown that it leads to a very limited amount of chaos.

10. A small amount of chaos is often beneficial

- In a chaos situation, the whole range of values are possible.
- Thus, the system goes over all possible values from this range in reasonable time.
- So, if we want to interact with the system, we can select the interaction moment at which the system is in the most beneficial state.
- This is a well-known phenomenon.
- For example, in construction, when a crane lifts some object to an upper floor level: this object does not stand still.
- It moves around somewhat in a chaotic way.
- In particular, after a few oscillations, the object gets the closest to the worker who needed this object.
- So the worker simply grabs the object when it is close.

11. A small amount of chaos is often beneficial (cont-d)

- Without such a chaotic motion:
 - delivering objects to upper floors
 - would require much more accurate (and thus, much more expensive and much more time-consuming) crane operations.
- Similarly, in economics, prices fluctuate with some degree of chaos.
- So a person who wants to sell something can naturally wait until the price is the highest within the given range.
- A person who needs the corresponding object can naturally wait until the price is the lowest within this range.
- The resulting benefit has the same origin as the benefit of investing money instead of spending them.
- You get a bonus since you sacrifice the ability to spend the money right away.

12. A small amount of chaos is often beneficial (cont-d)

- An interesting additional benefit of chaos appears in conflict situations.
- Chaos means that, in particular, that the adversary cannot predict the consequences of your move.
- This makes counter-measures less effective.
- For this purpose, additional chaos is added in situations ranging from soccer balls to military conflicts.

13. The presence of chaos necessitates the use of rule-based systems

- Traditionally, dynamics is described by systems of differential equations.
- Often, such equations enables us to make future predictions.
- However, as we have mentioned, sometimes:
 - small fluctuations of the initial conditions
 - lead to drastic changes after a moderately small time interval.
- So, we have an example of chaos.
- In many practical cases, we do not have a mathematical model describing the system's dynamics.
- We need to extract this model from the data.

14. The presence of chaos necessitates the use of rule-based systems (cont-d)

- In the chaos case:
 - we do not know the exact values of the future quantities, and
 - we do not know the exact values of the future probabilities.
- So often, we cannot use only the usual technique of differential equations to describe the system's dynamics.
- As the above construction example shows, in such cases, we need:
 - to supplement (or even replace) differential equations
 - with rules.
- In the construction example the corresponding rule was “If the object is close, grab it”.
- Such rules have been successfully used in control and other applications as part of the general fuzzy techniques.

15. The presence of chaos necessitates the use of rule-based systems

- Rule-based fuzzy techniques had many practical successes.
- This is an indication that the corresponding processes have a significant chaos component.
- Other examples of such indications are given in the following slides.

16. Sometimes, there is a direct evidence of chaos

- By our definition, chaos is the impossibility to predict:
 - either the exact values of future quantities
 - or at least the probability of different values.
- There are many situations when this impossibility is clear by itself.
- Let us mention three of them.

17. First example: modern AI

- The first example concerns the chaotic character of many real-life processes.
- It comes from the experience of Large Language Models (LLMs), the modern Artificial Intelligence (AI).
- These models use, in effect, probabilistic approach – in particular, they use it to predict future values and future probabilities.
- It is well known that in statistics, the more sample one uses for the analysis, the more accurate the predictions.
- In general, the prediction accuracy is proportional to $1/\sqrt{N}$, where N is the number of samples.
- LLMs use millions and billions of samples.
- So if the real-life processes were probabilistic, LLMs would predict the probabilities with accuracy of 99.9% or even higher.
- So the probability of error would be less than 0.1%.

18. First example: modern AI (cont-d)

- In reality, LLMs produce wrong answers in at least 5% of the cases.
- This is a clear indication that many real-life processes have a significant chaotic component.

19. Second example: human experts

- A similar example deals with human experts instead of AI.
- In 1974, a psychologist Paul Slovic checked:
 - how well experts can predict the future, and
 - how are their estimates of their success rate related to their actual success rate.
- Specifically, Slovic experimented with horse handicappers – experts that earn their living by predicting who will win in horse races.
- Slovic tested the experts on several situations in which the experts were provided with different amount of information.
- As we have mentioned, in general in statistics, the more information we have, the more accurately we can predict.
- People understand this.

20. Second example: human experts (cont-d)

- In particular, in Slovic's experiment:
 - the experts' degree of confidence in their answers steadily increased
 - when the amount of available information increased.
- However, interestingly, the actual percentage of correct answers stopped increasing.
- It stayed at the same 17% level once the experts were given 5 or more pieces of information.
- If the situation was purely probabilistic, the actual success rate would have increased – but it did not.
- This is an indication that the corresponding process is not purely probabilistic, that it has a significant chaos component.

21. Third example: stock market

- A similar example is specific for economy and finance – it is about the stock market.
- With numerous experts, the stock prices take into account future possible changes – at least for the major stocks.
- So, if the stock market was purely probabilistic, its predictions would become more and more accurate as the number of available data increases with time.
- In reality, there is always a discrepancy between the predicted and actual stock prices.
- This discrepancy does not decrease when new tools and new data become available.
- This shows that the stock market is not purely probabilistic, that it has a strong chaos component.

22. Indirect ways to detect chaos

- Probabilistic means, in particular, that:
 - in a sequence of values corresponding to different moments of time,
 - not only the arithmetic average of the first N elements of the sequence itself tends to the mean value,
 - but also that for any randomly or algorithmically selected subsequence, the average converges to the same value.
- This is what distinguishes truly random sequences from sequences like 0101....
- For 01..., which the average does tend to 0.5.
- However, for subsequences corresponding to odd and to even moments of time, averages are different.
- This phenomenon can be used to detect the presence of chaos.

23. Indirect ways to detect chaos (cont-d)

- Indeed, if a sequence is probabilistic, then:
 - the difference between the averages of two non-intersecting N -element subsequences is proportional to $\sqrt{2/N}$, and
 - for large N , this difference is approximately normally distributed.
- So:
 - if the average actual difference between such subsequences is larger than $k_0 \cdot \sqrt{2/N}$,
 - then – with the corresponding degree of confidence – we can conclude that there is a chaos component in this process.
- Here, as usual for normal distributions:
 - $k_0 = 2$ leads to 95% confidence,
 - $k_0 = 3$ leads to 99.9% confidence, and
 - $k_0 = 6$ leads to the confidence level $1 - 10^{-8}$.

24. Why do we need to process data and make decisions?

- What do we humans want?
- We want to use the predicted future values of different quantities $x_1, \dots, x_n$ to predict the values of related characteristics

$$y = f(x_1, \dots, x_n).$$

- In particular, we want to estimate the future value v of the appropriate objective function $v = J(x_1, \dots, x_n)$.
- Computing these values y and v is an important particular case of *data processing*.
- Instead of the exact future values, we may know the probability of different combinations $(x_1, \dots, x_n)$.
- In this case, we cannot determine the value y , but we would like to know the probability of different values of y .
- In particular, we want to find the probabilities of different values of v .

25. Why do we need to process data and make decisions (cont-d)

- Computing such probabilities is another case of data processing.
- In many situations, we can have some way to control the situation.
- In such situations, the values x_i depend on the parameters $c_1, \dots, c_k$ of our control action.
- The resulting value v of the objective function (or the resulting probability distribution) also depends on the control parameters c_i :

$$v = v(c_1, \dots, c_k).$$

- Sometimes, we can predict the exact values of the objective function.
- Then we want to select the control values c_i that lead to the largest possible value of the objective function.
- Selecting these values is an important particular case of *decision making*.

26. Why do we need to process data and make decisions (cont-d)

- Sometimes, we only know the probabilities of different values of the utility v .
- Then decision theory recommends to select the alternative for which the expected value of the utility is the largest.
- This is another case of decision making.

27. Chaos-related challenge

- In the presence of chaos, we cannot predict the future values x_i .
- We cannot even compute the probabilities of different combinations $(x_1, \dots, x_n)$.
- At best, we can predict the *range* $[\underline{x}_i, \overline{x}_i]$ of each value x_i .
- In the case of probabilities, we can predict the range $[\underline{p}, \overline{p}]$ of possible values of each corresponding probability.
- Probability distributions are often described by a cumulative distribution function (cdf) $F(x) \stackrel{\text{def}}{=} \text{Prob}(X \leq x)$.
- In the chaos case, for each x , we can only compute the range $[\underline{F}(x), \overline{F}(x)]$ of possible values of $F(x)$.
- Such an interval-valued cdf is known as a *probability box*, or, for short, *p-box*.
- How can we perform data processing and decision making under such uncertainty?

28. Data processing in the presence of chaos

- In the presence of chaos, for each of n inputs $x_1, \dots, x_n$, we only know the interval $[\underline{x}_i, \bar{x}_i]$ of possible values.
- In this case, the only thing that we can conclude about the quantity $y = f(x_1, \dots, x_n)$ is that this value y belongs to the range $[\underline{y}, \bar{y}]$ of possible values of $f(x_1, \dots, x_n)$:

$$[\underline{y}, \bar{y}] = \{f(x_1, \dots, x_n) : x_1 \in [\underline{x}_1, \bar{x}_1], \dots, x_n \in [\underline{x}_n, \bar{x}_n]\}.$$

- Computation of such a range is known as *interval computations*.
- There are many efficient interval computations algorithms.
- Often, the input intervals $[\underline{x}_i, \bar{x}_i]$ are narrow.
- This corresponds to the case when the chaos component is relatively small.

29. Data processing in the presence of chaos (cont-d)

- In this case, we can:
 - expand the function $y = f(x_1, \dots, x_n)$ in Taylor series around the midpoints $\tilde{x}_i \stackrel{\text{def}}{=} (\underline{x}_i + \bar{x}_i)/2$, and
 - ignore quadratic and higher terms in this expansion.
- In this case, we get $[\underline{y}, \bar{y}] = [\tilde{y} - \Delta, \tilde{y} + \Delta]$, where $\tilde{y} \stackrel{\text{def}}{=} f(\tilde{x}_1, \dots, \tilde{x}_n)$ and

$$\Delta = \sum_{i=1}^n \Delta_i \cdot \left| \frac{\partial f}{\partial x_i} \Big|_{x_1=\tilde{x}_1, \dots, x_n=\tilde{x}_n} \right|, \text{ where } \Delta_i \stackrel{\text{def}}{=} \frac{\bar{x}_i - \underline{x}_i}{2}.$$

- Algorithms are also known for the case of p-box uncertainty.

30. Decision making in the presence of chaos: interval case

- Which control $c = (c_1, \dots, c_k)$ should we select if:
 - for each c ,
 - we only know the interval $[\underline{v}(c), \bar{v}(c)]$ of possible values of the objective function?
- In such cases, decision theory recommends to select the control c for which the combination $\alpha \cdot \bar{v}(c) + (1 - \alpha) \cdot \underline{v}(c)$ is the largest possible.
- The value α should be selected by the decision maker depending on his/her level of optimism/pessimism.

31. Decision making in the presence of chaos: interval case (cont-d)

- Specifically:
 - when $\alpha = 1$, the decision maker only takes into account the best-case (= perfectly optimistic) situation;
 - when $\alpha = 0$, the decision maker only takes into account the worst-case (= perfectly pessimistic) situation;
 - when α is between 0 and 1, the decision maker takes into account both the best-case and the worst-case scenarios.
- The use of the above α -combination was first justified by the economist Leo Hurwicz.
- Hurwicz later received a Nobel Prize for this research.
- In his honor, this technique is known as *Hurwicz optimism-pessimism criterion*.

32. Decision making in the presence of chaos: case of imprecise probabilities

- Sometimes, the uncertainty is described by imprecise probabilities (e.g., by a p-box).
- This means that several different probability distributions are consistent with the available information.
- For each possible probability distribution, we can compute the expected value of the utility.
- These expected values form an interval $[\underline{v}(c), \bar{v}(c)]$.
- We can then use Hurwicz's criterion to select the control c .

33. Acknowledgments

This work was supported by:

- the AT&T Fellowship in Information Technology,
- the Institute for Risk and Reliability, Leibniz Universitaet Hannover, Germany,
- the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Focus Program SPP 100+ 2388, Grant Nr. 501624329,
- the European Union under the project ROBOPROX (No. CZ.02.01.01/00/22 008/0004590),
- the Center of Excellence in Econometrics, Faculty of Economics, Chiang Mai University, Thailand,
- the Ho Chi Minh City University of Banking, Vietnam, and
- Thang Long University, Hanoi, Vietnam.