How Earthquake Risk Depends on the Closeness to a Fault: Symmetry-Based Geometric Analysis

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1. Earthquake Prediction Is an Important Problem

- Earthquakes can lead to a huge damage – and the big problem is that they are very difficult to predict.
- To be more precise, it is very difficult to predict the time of a future earthquake.
- However, we *can* estimate which earthquake locations are probable.
- In general, earthquakes are mostly concentrated around the corresponding faults.
- For some faults, all the earthquakes occur in a narrow vicinity of the fault.
- For others, areas more distant from the fault are risky as well.
- To properly estimate the earthquake's risk, it is important to understand:
  - when this risk is limited to a narrow vicinity of a fault and
  - when this risk is not thus limited.

2. Case Study: San Andreas Fault

- This problem has been thoroughly studied for the most well-studied fault in the world: San Andreas fault.
- This fault consists of somewhat different Northern and Southern parts.
- The Northern part is close to a straight line.
- In this part, the fault itself is narrow – e.g., it is less than a mile wide in the Olema Trough part.
- Earthquakes are mostly limited to a narrow vicinity of this line, within ± 10 miles.
- The Southern part is geometrically different: it is curved.
- In the South, the fault itself is much wider – e.g., it is many miles across in the Salton Trough part.
- Earthquakes can happen much further from the main fault, at a distance up to 30 miles away.

3. Resulting Problem

- It would be great to find a general explanation for this phenomenon.
- This will help us better understand other, not so well-studied faults.
- In this paper, we show that the above phenomenon has a general geometric explanation.
- It can be, thus, probably be extended to other faults as well.
- In this research, we will be using the idea of *symmetries*.
- Symmetries is one of the fundamental – and one of the most successful – ideas in physics in genera.
- However, the idea of symmetries is not yet as popular – and even not yet well known – in engineering and geosciences.
- So, we need to explain this idea in some detail.

4. Why Symmetries

- The idea of symmetry comes from the way we make predictions.
- For example, if you have a pen in your hand and you drop it, it will fall down with the acceleration of 9.81 m/sec^2.
- If you rotate yourself by 90 degrees and repeat the same experiment, you will get the same result.
- You can rotate yourself by other angles – and still get the same results.
- So, after several such experiments, you can reasonably confidently conclude that:
  - the pen-falling-down process does not change
  - if we simply rotate the whole setting by any angle.
- Similarly, if you step a few steps in any direction, and repeat the same pen-falling-down experiment, you will get the same result.
- If you repeat this experiment in Hannover, Germany, instead of El Paso, Texas, the result will be the same.

5. Why Symmetries (cont-d)

- Let us ignore for now the minor difference in the gravitational fields.
- This difference is minor for the purpose of this experiment but it provides very important geophysical information.
- Thus, we can conclude that the results of the experiment do not change if we shift the experiment to a different location.
- This is how we, in general, make predictions.
- We observe that some phenomenon does not change if we perform some changes ("transformations") to its setting.
- Then, we can conclude that in the future, if we perform a similar transformation, we should get the same result.
- The experiments do not have to be as simple as dropping a pen.
- For example, how do we know that Ohm's law – according to which the voltage V is proportional to the current I – is valid?

6. Why Symmetries (cont-d)

- Ohm observed it in Denmark.
- Then different researchers observed the exact same phenomenon in different locations.
- So now we can conclude that this law is indeed universally valid.
- The symmetries also do not have to be as simple as rotations and shifts.
- For example, in engineering, many processes do not change if we change the scale.
- That is why testing a small-size model of a plane helped us to understand how the actual full-size plane will behave in flight.
- In physics, there are even more complex examples of symmetries
- For example, if we replace elementary particles by the corresponding antiparticles, almost all physical processes will remain the same.
- If we invert the flow of time, most equations remains valid, etc.

7. What Is Symmetry: Towards a Formal Definition

- To describe what is symmetry, we need to have a class of possible transformations – rotations, shifts, particle $\to$ antiparticle.
- If two different transformations T_1 and T_2 are possible, then we can first perform the first one and then the second one.
- Thus, we get a combined transformation T_2T_1 which is called a *composition*.
- We can have a composition of more than two transformations: e.g., if we first apply T_1, then T_2, and then T_3, then we get a composition $T_3T_2T_1$.
- It is easy to see that we get the same process:
  - whether we first apply T_2T_1 and then T_3, or
  - whether we first apply T_1, and then T_3T_2:

$$T_3(T_2T_1) = (T_2T_2)T_1.$$

- In mathematical terms, this means that the composition operation is *associative*.

8. Towards a Formal Definition (cont-d)

- Also, most transformation are *reversible.*
- If we rotate by 90 degrees to the right, we can then rotate by 90 degrees to the left and thus come back to the original position.
- If we go forward 10 meters, we can then go back 10 meters and thus come back to the original position.
- If we replace each particle with its antiparticle, we can then repeat the same replacement and get back the original matter, etc.
- This "reversing" transformation – denoted by T^{-1} – has the property that it cancels the effect of the original one: $T^{-1}T = TT^{-1} = I$.
- Here, I is the "identity" transformation that does not change anything.
- For the identity transformation, we have $TI = IT = T$ for all T.

9. Towards a Formal Definition (cont-d)

- So, on the class of all transformations, we have an associate binary operation for which:
  - there is a transformation I for which $TI = IT = T$ for all T, and
  - for each T, there is an "inverse" T^{-1} for which $T^{-1}T = TT^{-1} = I$.
- In mathematics, a pair consisting of a set and a binary operation with these properties is called a *group*.
- Thus, possible transformations form a group.
- This group is usually called a *transformation group*.

10. How Physical Laws Are Described in These Terms

- As we have mentioned, many physical laws simply mean that a certain property does not change under some class of transformations.
- In mathematical terms, we can say that that these properties are *invariant* under the corresponding transformation groups.
- In physics, transformations for which some properties are preserves are also called *symmetries*.
- The corresponding transformation group is called a *symmetry group*.
- These terms are consistent with the usual meaning of the word "symmetry".
- E.g., when we say that a football is spherically symmetric, we mean that its shape does not change if we rotate it in any way around its center.
- In this case, rotations are symmetries of this ball.

11. This Approach Has Been Very Successful in Physics

- In the past – starting with Isaac Newton – new physical theories were usually described in terms of differential equations.
- However, starting from the 1960s quark theory, many physical theories are now formulated exclusively in terms of symmetries.
- Then, equations follow from these symmetries.
- Moreover, it turned out that:
  - many classical physical theories that were originally formulated in terms of differential equations,
  - can be derived from the corresponding symmetries.
- Symmetries can help not only to explain theories, but to explain phenomena as well.
- For example, there are several dozens theories explaining the spiral structure of many galaxies – including our Galaxy.
- It has been shown that all possible galactic shapes – and many other physical properties – can be explained via symmetries.

12. Symmetries Beyond Physics

- Similarly, symmetries can be helpful in biology – where they explain, e.g., Bertalanfi equations describing growth.
- Symmetries have been helpful in computer science – when they help with testing programs, and in many other disciplines.
- Symmetries not only explain, they can help design.
- For example, symmetries (including non-geometric ones like scalings) can be used to find an optimal design for a network of radiotelescopes.
- Symmetries can help to come up with optimal algorithms for processing astroimages.
- Natural symmetries can also explain which methods of processing expert knowledge work well and which don't.

13. What Are the Symmetries in Our Problem?

- On a very large scale, the Earth's geophysical structure is reasonably homogeneous and isotropic.
- So, in the first approximation, each piece of the Earth surface is symmetric with respect to shifts and rotations.
- We also do not have any selected distances.
- This means that the initial configuration is invariant with respect to *scalings* $x_i \to \lambda \cdot x_i$ corr. to changing the unit of distance.

14. The Corresponding Symmetry Is Unstable

- In the ideal case, the Earth would be perfectly symmetric, it will have the exact same properties at each geographic location.
- In particular, we will have the molten material at exactly the same depth at each location.
- However, as geophysicists know, this complete symmetry is unstable.
- E.g., due to random fluctuations, at some location, magma penetrates higher than in other locations.
- So in this location, the barrier for magma becomes thinner and thus, easier to penetrate.
- As a result, the magma from the surrounding areas start flowing into this area and push up even more.
- So, the initially small perturbation grows and grows – until the magma comes to the Earth's surface as lava from a newborn volcano.
- Such increase in asymmetry is ubiquitous in physics, it is known as *spontaneous symmetry breaking*.

15. Which Spontaneous Symmetry Breakings Are Most Probable

- According to statistical physics, the most probable are symmetry breakings that retain the largest number of symmetries.
- This may sound like a very abstract and not very intuitive idea, but many examples of it are very intuitive.
- For example, at low temperatures, every material becomes a crystal, i.e., has many symmetries.
- In the liquid state, there are fewer symmetries: e.g., volume is preserved but not much else.
- Finally, in the state of gas, there are, in effect, no symmetries at all.
- And indeed, the transition from one state to another follows the above general idea; when heated, a solid body:
  - usually turns first into liquid (i.e., state with some symmetries)
  - and not directly into gas (state with no symmetries).
- Let us apply this principle to our situation as well.

16. What Are the Resulting Shapes

- We start with a state which is invariant under arbitrary shifts, rotations, and scalings.
- After the spontaneous symmetry breaking, according to the above physical principle, the most probable state will still have some symmetries.
- Let us denote the corresponding symmetry group by G.
- This remaining symmetry means that:
  - if we have a perturbation at some location a,
  - then, for each transformation $g \in G$, we will have a similar perturbation at the location $g(a)$ obtained from a by applying g.
- Thus, together with each a, the set of all locations where we observe a similar perturbation contains the whole set $G(a) \stackrel{\text{def}}{=} \{g(a) : a \in G\}$.
- In mathematical terms, this set is called an *orbit*.
- Thus, we can conclude that the resulting shape consists of orbits of the remaining symmetry group G.

17. Resulting Shapes

- It is easy to show that each orbit S is itself invariant with respect to the symmetry group G.
- For the group of all shifts, rotations, and scalings on the plane, all subgroups and corresponding orbits are well-known.
- When the group is large enough – e.g., if it contains all shifts – the orbit is the whole plane.
- The only connected orbits which are different from the whole plane are:
  - straight lines, half-lines,
  - circles, and logarithmic spirals $\ln(r) = p + q \cdot \varphi$.
- Indeed, faults are either almost straight lines or curves – shaped like segments of circles or segments of logarithmic spirals.
- Of course, we are only talking about a local shape: a straight line goes all the way to infinity, but a fault is usually a reasonably local phenomenon.

18. Which Fault Shape Should Be More Frequent

- In case of a straight line, we have a 2-D remaining symmetry group:
  - a straight line is invariant with respect to shifts along this line, and
  - it is also invariant with respect to scalings.
- In contrast, circles and logarithmic spirals only have a 1-D symmetry groups:
  - a half-line is invariant with respect to scalings,
  - a circle is invariant with respect to all rotations around its center, and
  - a logarithmic spiral is invariant with respect to combined rotation-and-scaling transformations $\varphi \to \varphi + \varphi_0$, $r \to \exp(q \cdot \varphi_0) \cdot r$.
- Thus, straight-line faults should be more probable – and thus, more frequent – that the curved-shaped ones.
- And indeed, in nature, most faults are close to straight lines, and curves faults are much more rare.

19. Shape of The Near-Fault Earthquake Activity Area

- Let us use the above results to analyze the original problem: what is the shape of the near-fault seismically active area.
- In our analysis, we used the idea that the system should be invariant with respect to some subgroup of the original symmetry group.
- We used this idea to derive possible fault shapes, and we concluded that we have four options:
  - a straight line (with shifts and scalings),
  - a half-line (with scalings only),
  - a circle (with rotations), and
  - a logarithmic spiral (with combined rotation-and-scaling symmetries).
- It is reasonable to conclude that the near-fault earthquake-prone risk region should have the same symmetries as the fault.

20. Near-Fault Activity Area (cont-d)

- For a straight line, these symmetries are shift and re-scaling.
- The only region with the same symmetries is the fault itself.
- This explains why there is practically no activity outside the fault.
- For half-line, i.e., for a fault with an abrupt end-point – an angular segment has the same symmetries.
- Similarly, for a circle or for a logarithmic spiral, if we start with a different point, we can have another orbit with the same symmetries.
- For example, a circular disk has the same symmetries as the circle. Thus, for faults of this shape, earthquakes outside the fault are possible.

21. Conclusion

- For the San Andreas fault, it was observed that:
  - for the continuous straight-line fault segment, only a very narrow vicinity of a fault is risk-prone – ≤ 10 miles from the fault, while
  - for the curved-shaped fault segment, earthquakes can also happen at a reasonable distance from the fault, up to 30 miles distance.
- In this talk, we show that this empirical phenomenon has a solid geometric explanation.
- Thus, we expect that the same phenomenon will be observed at other faults as well.

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