How to Deal with Context in Computing with Words: A Type-2-Motivated Approach

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1. Type-2-Motivated Approach

- An expert describes his/her opinion about a quantity by using imprecise ("fuzzy") natural language words.
- Example: "small", "large", etc.
- Each of these words provides a rather crude description of the corresponding quantity.
- A natural way to refine this description is to assign degrees to which the observed quantity fits each word.
- For example, an expert can say that the value is reasonable small, but to some extent it is medium.
- In this refined description, we represent each quantity by a tuple of the corresponding degrees.

2. Challenge

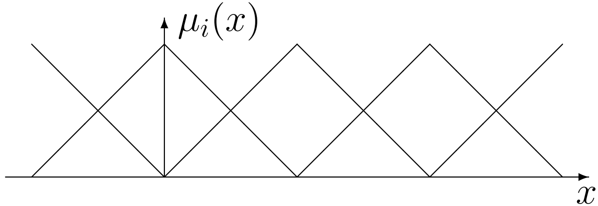
- Need for data processing:
  - we have such a tuple-based information about several quantities $x_1, \ldots, x_m$, and
  - we know that another quantity y is related to x_i by a known relation $y = f(x_1, \ldots, x_m)$;
  - it is desirable to come up with a resulting tuple-based description of y.
- It turns out that a seemingly natural idea for computing such a tuple does not work.
- This idea cane be modified so that it can be used.

3. How to Implement the Above Approach

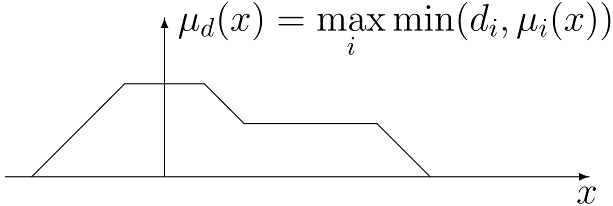
- We have degree d_i assigned to the i-th word, with membership function $\mu_i(x)$.
- Based on this information, what is then the degree $\mu_d(x)$ to which x is a possible value?
  - either the quantity is described by the 1st word, and this word is adequate for x; degree $\min(d_1, \mu_1(x))$;
  - here, we interpret "and" as min;
  - or the quantity is described by the 2nd word, and this word is adequate for x; the degree $\min(d_2, \mu_2(x))$;
  - etc.
- We interpret "or" as *max*.
- So, the resulting degree is $\mu_d(x) = \max_i \min(d_i, \mu_i(x))$.

4. How to Implement the Above Approach (cont-d)

- Simplest membership functions:



- The resulting degree $\mu_d(x) = \max\limits_i \min(d_i, \mu_i(x))$:

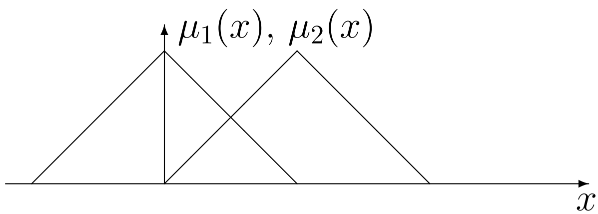


5. A Seemingly Natural Implementation

- We want to be able to transform a general membership function $\mu(x)$ into a tuple of degrees $d = (d_1, \ldots, d_n)$.
- Our hope is that for the f-n $\mu_d(x) = \max\limits_i \min(d_i, \mu_i(x))$, we get back the degrees d_i.
- Seemingly natural idea: $\mu(x)$ corresponds to the i-th word if:
  - either a value x is in agreement with $\mu(x)$ and with this word;
  - or a value x' is in agreement with $\mu(x)$ and with this word;
  - etc.
- The resulting degree is $\max\limits_x \min(\mu(x), \mu_i(x))$.

6. Limitations of a Seemingly Natural Implementation

- *Idea:* estimate d_i as $\max\limits_x \min(\mu(x), \mu_i(x))$.
- Our hope is that for the f-n $\mu_d(x) = \max\limits_i \min(d_i, \mu_i(x))$, we get back the degrees d_i.
- *Problem:* for the basic function $\mu(x) = \mu_1(x)$ corr. to $d = (1, 0 \ldots, 0)$, we do not get back $(1, 0, \ldots, 0)$:



- Specifically, for $\mu(x) = \mu_1(x)$, we get

$$\max_x \min(\mu(x), \mu_2(x)) = 0.5 \neq d_2 = 0.$$

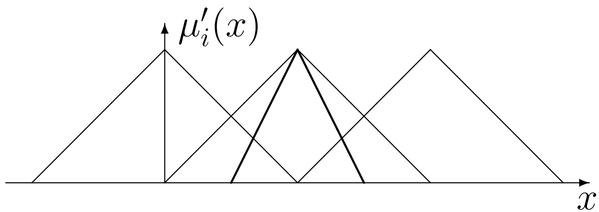
7. Towards a Possible Solution

- Intersection leads to $\max\limits_x \min(\mu(x), \mu_i(x)) \neq d_i$.
- So let us remove the intersecting parts from the membership function before applying the above formula:
  - we compute "reduced" basic functions
  $$\mu'_i(x) = \max(0, \mu_i(x) - \max(\mu_{i-1}(x), \mu_{i+1}(x)));$$
  - we also compute the "reduced" membership function
  $$\mu'(x) = \max(0, \mu(x) - \max(\mu_{i-1}(x), \mu_{i+1}(x)));$$
  - then, we compute the degrees based on these reduced functions, as
  $$\widetilde{d}_i = \max_x(\min(\mu'(x), \mu'_i(x))).$$

8. This Idea Does Lead to a Possible Solution

- *Reminder:* we compute $\widetilde{d}_i = \max_x(\min(\mu'(x), \mu_i'(x)))$, where

$$\mu_i'(x) = \max(0, \mu_i(x) - \max(\mu_{i-1}(x), \mu_{i+1}(x)));$$



$$\mu'(x) = \max(0, \mu(x) - \max(\mu_{i-1}(x), \mu_{i+1}(x)));$$

- Interesting fact: if we apply this to the function $\mu_d(x) = \max\limits_i \min(d_i, \mu_i(x))$, we do get back the degrees d_i:

$$\widetilde{d}_i = d_i.$$