Towards Combining Probabilistic, Interval, Fuzzy Uncertainty, and Constraints: An Example Using the Inverse Problem in Geophysics

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1. Cyberinfrastructure: What It Does Now

- *Situation:*
  - large amounts of data are stored in different locations;
  - algorithms for processing this data are also implemented at different locations;
  - the ability to use the data and algorithms is not shared
- *Traditional solution:* centralization.
- *Drawback:* excessive workload.
- *New solution:* web services (cyberinfrastructure) allow users to submit requests without worrying about the geographic locations of different computational resources (databases and programs).
- Web services enable the user to receive:
  - the desired data $x_1, \ldots, x_n$ and
  - the results $y = f(x_1, \ldots, x_n)$ of processing this data.

2. Cyberinfrastructure: What It Should Do

- *Known fact:* data x_i usually come from measurements.
- *Uncertainty:* in general, the measured values $\widetilde{x}_i$ are different from the actual (unknown) values x_i:
$$\widetilde{x}_i \neq x_i$$
- *Result:* the result $\widetilde{y}$ of data processing is, in general, different from the actual value y of the desired quantity:
$$\widetilde{y} = f(\widetilde{x}_1, \ldots, \widetilde{x}_n) \neq y = f(x_1, \ldots, x_n).$$
- *Problem:* gauge this difference.

3. Combining Uncertainty Models Within Cyberinfrastructure – A Challenging Problem

- *The main objective of cyberinfrastructure* is to be able:
  - to seamlessly move data between different databases

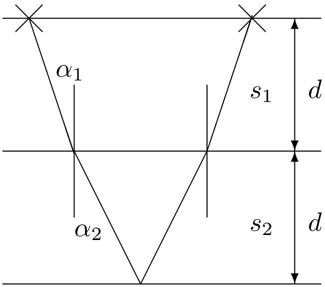
    (where this data is stored in different formats),
  - to feed the combined data into a remotely located program

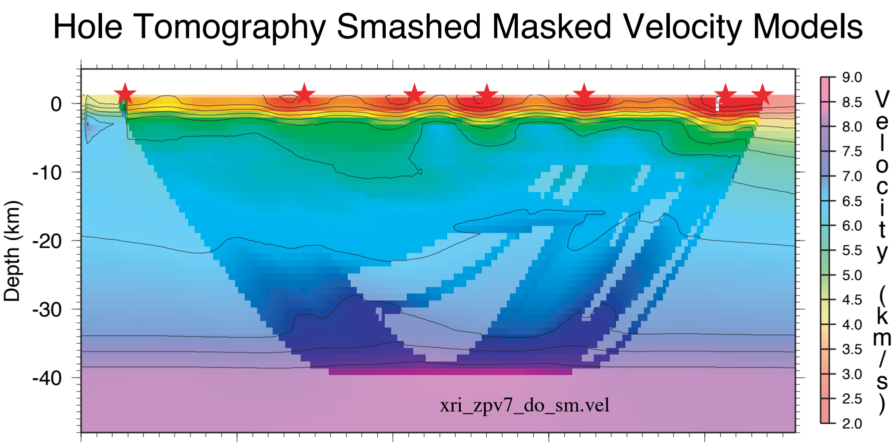
    (which may require yet another data format),

    and
  - to return the result to the user.
- *Additional objectives of cyberinfrastructure:* it is also important to gauge the quality and accuracy of the result.
- *Situation:* we may have different models for describing uncertainty of different databases and programs.
- *Conclusion:* it is important to be able to combine different uncertainty models.

4. Case Study: The Seismic Inverse Problem

- *We know:* traveltimes x_i between different locations (based on passive or active seismic measurements).
- *We would like to find:* slowness values.
- *Most widely used method:* Hole's tomographic inversion code
  - we start with the initial slowness model $s_j^{(0)}$;
  - based on the current model, we find the ray paths;
  - based on the ray paths, we predict the traveltimes $t_i = \sum\limits_j \ell_{ij} \cdot \Delta s_j$;
  - based on the differences $\Delta t_i \stackrel{\text{def}}{=} \widetilde{t}_i - t_i$, we adjust the slownesses: $s_j^{(k+1)} = s^{(k)} + \Delta s_j$, where Δs_j is the average of $\Delta s_{ij} \stackrel{\text{def}}{=} \dfrac{\Delta t_i}{L_i}$,
  - repeat until $\dfrac{1}{n} \cdot \sum (\Delta t_i)^2 \le \sigma^2$.





Hole Tomography Smashed Masked Velocity Models
Depth (km)
0
-10
-20
-30
-40
xri_zpv7_do_sm.vel
Velocity (km/s)
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0

5. Seismic Inverse Problem: Remaining Problems and Possible Solutions

- *Room for improvement:*
  - the resulting model is sometimes not very accurate;
  - it is often not clear how accurate the resulting model is;
  - the convergence is often slow – sometimes, requires time-consuming manual adjustment of the initial model;
  - the inversion is often unstable because known features are not included in the starting model.
- *Natural solution:*
  - The existing techniques only take into account the measurement results.
  - Expert geophysicists have prior knowledge about the structure of the region.
  - This additional knowledge can:
    * improve the accuracy of the geophysical structures;
    * speed up convergence.

6. Probabilistic Prior Knowledge

- *Measurement uncertainty:* measurement results often come with probabilistic uncertainty.
- *Probabilistic uncertainty:* we know the probability distribution $\rho_i(\Delta x_i)$ of different values $\Delta x_i \stackrel{\text{def}}{=} \widetilde{x}_i - x_i$ of measurement errors.
- *It is possible to use this probabilistic uncertainty:* as a result of processing such measurement results, we usually obtain
  - not only the estimate $\widetilde{y}$ of the desired quantity, but also
  - the standard deviation σ (and other statistical characteristics) of possible differences $\widetilde{y} - y$ between the estimated $\widetilde{y}$ and the actual y values of the desired quantity.
- *Some prior knowledge comes from measurements:* prior knowledge sometimes comes from previous measurement processing.
- *Probabilistic form of prior knowledge:* such prior knowledge has the form

$$y \approx y^{(0)}, \text{ with standard deviation } \sigma^{(0)}.$$

- *Probabilistic prior knowledge has been used:* this prior knowledge has been successfully used, e.g., in tomography of Central Asia.

7. The Need to Use Interval and Fuzzy Prior Knowledge

- *Prior knowledge as bounds:* in some situations, we have a different type of prior knowledge.
- *Example:* a geophysicist may know that the speed of sound at a certain depth must be between 6 and 8 km/s.
- *Natural idea:* assume that 7 is the most probable value and 1 is the standard deviation.
- *Problem:* the expert does not necessarily consider any value within the interval [6,8] to be more probable than others.
- *Alternative idea:* assume that the distribution is uniform.
- *Problem:* the probability to have a velocity close to 6 at all cells is very small, so statistical approach will dismiss it.
- However, from the geophysicist viewpoint, it is quite possible that $v \approx 6$ for all the cells.
- *New idea:* explicitly keep the interval as the prior data.
- *Fuzzy uncertainty:*
  - in addition to an interval that is guaranteed to contain the slownesses,
  - the expert can often produce several nested intervals corresponding to different degrees of certainty.

8. How We Can Use Interval Uncertainty

- *How algorithms work now:*
  - start with a reasonable velocity model;
  - predict traveltimes t_i between stations;
  - use the difference $\Delta t_i \stackrel{\text{def}}{=} \widetilde{t}_i - t_i$, where $\widetilde{t}_i$ are measured values, to adjust the velocity model:
    * divide Δt_i by the length L_i of the ray path;
    * add the $\Delta t_i / L_i$ to all slownesses along the ray path;
    * for each cell, take an average of the adjusted slownesses corresponding to different ray paths.
- *How to modify* when we know the interval $[\underline{s}_j, \overline{s}_j]$ of possible slownesses:
  - first, we compute the next approximation $s_j^{(k+1)}$ to the slownesses,
  - then, we replace $s_j^{(k+1)}$ with the nearest value within the interval $[\underline{s}_j, \overline{s}_j]$:
    * we take $s_j = \overline{s}_j$ if $s_j^{(k+1)} > \overline{s}_j$, and
    * we take $s_j = \underline{s}_j$ if $s_j^{(k+1)} < \underline{s}_j$.
- *Problem:* we do not get fully compensate Δt_i.
- *Better idea:* we distribute the whole difference Δt_i between cells in such a way that all resulting slownesses are within $[\underline{s}_j, \overline{s}_j]$.

9. How Accurate Is The Resulting Velocity Model

- *Sources of uncertainty:*
  - measurement errors in measuring traveltimes;
  - incomplete coverage.
- *First idea:* use standard statistical techniques (e.g., Monte-Carlo simulations).
- *For passive seismic data:* reasonable error bounds.
- *For active seismic data:* since error decreases as $\sim 1/\sqrt{n}$, where n is the number of measurements, we get accuracy 10 m/sec at 30 km depth.
- *Explanation:*
  - we implicitly assumed that all measurement errors are independent, but
  - all measurements may have the same systematic error component.
- *Natural idea:* worst-case analysis, without any independence assumptions.
- *Problem:* too pessimistic error estimates.
- *Combined idea:*
  - *bootstrap:* comparing results from the two halves of data;
  - *checkerboard:* determines the size of the cells about which we make accuracy estimates.

10. Work in Progress

- *Real-life situations:*
  - some prior knowledge comes from prior data processing,
  - some prior knowledge comes from prior interval constraints.
- *Conclusion:* we must combine different types of uncertainty.
- *The ultimate goal:* to provide visual presentations of the combined effects of different types of uncertainty.

11. Acknowledgments

This work was supported in part:

- by NSF grant EAR-0225670,
- by NASA under cooperative agreement NCC5-209,
- by NIH grant 3T34GM008048-20S1,
- by Army Research Lab grant DATM-05-02-C-0046,
- by Star Award from the University of Texas System,
- and by Texas Department of Transportation grant No. 0-5453.