Grading Homeworks, Verifying Code: How Thorough Should the Feedback Be?

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1. How Thoroughly Should We Grade the Homeworks?

- In the ideal world, we should assign many homeworks, and grade each of them very thoroughly.
- In reality, our time – and if we have a Teaching Assistant (TA), the time of the TA – is limited.
- So, we have a choice:
  - we can assign few homeworks and grade all of them very thoroughly,
  - or we can assign many homeworks and grade all of them less thoroughly.
- What is the optimal level of thoroughness?

2. Comment

- Doing homeworks helps students, even if these homeworks are not graded at all.
- This provides the students with additional practice.
- There are many ways to entice the students to do homeworks even when not all homeworks will be graded.
- We can assign several problems, but grade only some.
- The selection of graded problems can be decided by a random choice after the homeworks are submitted.
- So the students do not know beforehand which problems will be graded.
- Another idea is, after the homework is submitted, to randomly select students whose homeworks will be graded.
- It's also possible to combine these two ideas.

3. How Thoroughly Should We Verify the Code

- Usually, when a company designs software, the resulting code is not only tested to make sure it works.
- It is also verified by specialists who make sure that this code is well-written; in the ideal world:
  - every line of code should be thoroughly analyzed and verified, and
  - the detailed feedback should be provided to the programmer.
- In practice, the verifier's time is limited, so:
  - we can verify fewer lines of code and provide more thorough feedback,
  - or we can verify more lines of code and provide less thorough feedback.
- What is the optimal level of thoroughness?

4. Simplified Case, When We Ignore the Difference Between Students

- Let us first consider the simplified case, when:
  - we ignore the difference between students, and
  - assume that the effect of feedback is the same for all students.
- Let us denote the effect of a piece of feedback that took t hours by $f(t)$.
- Let T denote the overall time allocated to producing this feedback.
- What we want is find the value n and the values $t_1, \ldots, t_n$ for which $t_1 + \ldots + t_n = T$.
- We want to maximize the overall effect

$$E = f(t_1) + \ldots + f(t_n).$$

5. Solving the Problem

- Lagrange multiplier technique reduces our problem to the unconstrained optimization problem of maximizing

$$f(t_1) + \ldots + f(t_n) + \lambda \cdot (t_1 + \ldots + t_n - T).$$

- Differentiating this expression with respect to t_i and equating the derivative to 0, we get $f'(t_i) + \lambda = 0$.
- So, in general, the values t_i should be the same:

$$t_1 = \ldots = t_n, \text{ so } n \cdot t_1 = T \text{ and } E = n \cdot f(t_1).$$

- Thus, $n = \dfrac{T}{t_1}$ and $E = \dfrac{T}{t_1} \cdot f(t_1)$.
- So, the optimal t_1 should maximize the expression

$$\frac{E}{T} = \frac{f(t_1)}{t_1}.$$

- This is equivalent to equating the derivative of this expression to 0, which leads to $f'(t_1) \cdot t_1 = f(t_1)$.

6. Case of Empirical Dependence

- Empirically: $f(t) = 1 - \dfrac{c}{t^q + c}$.
- For this dependence, the optimal time is

$$t_1 = (c \cdot (q-1))^{1/q}.$$

- This optimal time does not depend on the overall time T.
- This means that if we get extra time – e.g., get a TA:
  - we should increase number of homeworks,
  - but we should spend the same time grading each homework.

7. General Case, When We Take Into Account That Students Are Different

- In practice, the effect function $f_i(t)$ is different for different students.
- To be fair, we divide the grading time T equally between all s students.
- Then, for each student i, the optimal time t_i for grading one homework can be found from the formula

$$f_i'(t_i) \cdot t_i = f_i(t_i).$$

- For the empirical formula $f_i(t) = \dfrac{t^{q_i}}{t^{q_i} + c_i}$, we get

$$t_i = (c_i \cdot (q_i - 1))^{1/q_i}.$$

8. General Case (cont-d)

- The optimal times t_i are different for different students.
- However, we allocated the same overall grading time $\frac{T}{s}$ for each student.
- This means that different students should get different number of homeworks:
  - to some students, we assign fewer homeworks, and provide more thorough feedback, while
  - to other students, we assign more homeworks, and provide less thorough feedback.

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