Rotation-Invariance Can Further Improve State-of-the-Art Blind Deconvolution Techniques

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1. Image Deconvolution: Formulation of the Problem

- The measurement results y_k differ from the actual values x_k dues to additive noise and blurring:

$$y_k = \sum_i h_i \cdot x_{k-i} + n_k.$$

- From the mathematical viewpoint, y is a *convolution* of h and x: $y = h \star x$.
- Similarly, the observed image $y(i,j)$ differs from the ideal one $x(i,j)$ due to noise and blurring:

$$y(i,j) = \sum_{i'} \sum_{j'} h(i-i', j-j') \cdot x(i', j') + n(i,j).$$

- It is desirable to reconstruct the original signal or image, i.e., to perform *deconvolution*.

2. Ideal No-Noise Case

- In the ideal case, when noise $n(i,j)$ can be ignored, we can find $x(i,j)$ by solving a system of linear equations:

$$y(i,j)=\sum_{i'}\sum_{j'} h(i-i',j-j')\cdot x(i',j').$$

- However, already for 256×256 images, the matrix h is of size 65,536×65,536, with billions entries.
- Direct solution of such systems is not feasible.
- A more efficient idea is to use Fourier transforms, since $y=h\star x$ implies $Y(\omega)=H(\omega)\cdot X(\omega)$; hence:
  - we compute $Y(\omega)=\mathcal{F}(y)$;
  - we compute $X(\omega)=\dfrac{Y(\omega)}{H(\omega)}$, and
  - finally, we compute $x=\mathcal{F}^{-1}(X(\omega))$.

3. Deconvolution in the Presence of Noise with Known Characteristics

- Suppose that signal and noise are independent, and we know the power spectral densities

$$S_I(\omega) = \lim_{T\to\infty} E\left[\frac{1}{T}\cdot|X_T(\omega)|^2\right], S_N(\omega) = \lim_{T\to\infty} E\left[\frac{1}{T}\cdot|N_T(\omega)|^2\right]$$

- We minimize the expected mean square difference

$$d \stackrel{\text{def}}{=} \lim_{T\to\infty} \frac{1}{T}\cdot E\left[\int_{-T/2}^{T/2} (\widehat{x}(t)-x(t))^2\,dt\right].$$

- Minimizing d leads to the known Wiener filter formula

$$\widehat{X}(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 + \dfrac{S_N(\omega_1,\omega_2)}{S_I(\omega_1,\omega_2)}}\cdot Y(\omega_1,\omega_2).$$

4. Blind Image Deconvolution in the Presence of Prior Knowledge

- Wiener filter techniques assume that we know the blurring function h.
- In practice, we often only have partial information about h.
- Such situations are known as *blind deconvolution*.
- Sometimes, we know a joint probability distribution $p(\Omega, x, h, y)$ corresponding to some parameters Ω:

$$p(\Omega, x, h, y) = p(\Omega) \cdot p(x|\Omega) \cdot p(h|\Omega) \cdot p(y|x, h, \Omega).$$

- In this case, we can find

$$\widehat{\Omega} = \arg\max_{\Omega} p(\Omega|y) = \int\int_{x,h} p(\Omega, x, h, y)\, dx\, dh \text{ and}$$

$$(\widehat{x}, \widehat{h}) = \arg\max_{x,h} p(x, h|\widehat{\Omega}, y).$$

5. Blind Image Deconvolution in the Absence of Prior Knowledge: Sparsity-Based Techniques

- In many practical situations, we do not have prior knowledge about the blurring function h.
- Often, what helps is *sparsity* assumption: that in the expansion $x(t) = \sum_i a_i \cdot e_i(t)$, most a_i are zero.
- In this case, it makes sense to look for a solution with the smallest value of

$$\|a\|_0 \stackrel{\text{def}}{=} \#\{i : a_i \neq 0\}.$$

- The function $\|a\|_0$ is not convex and thus, difficult to optimize.
- It is therefore replaced by a close *convex* objective function $\|a\|_1 \stackrel{\text{def}}{=} \sum_i |a_i|$.

6. State-of-the-Art Technique for Sparsity-Based Blind Deconvolution

- Sparsity is the main idea behind the algorithm described in (Amizic et al. 2013) that minimizes

$$\frac{\beta}{2}\cdot\|y-\mathbf{W}a\|_2^2+\frac{\eta}{2}\cdot\|\mathbf{W}a-\mathbf{H}x\|_2^2+\tau\cdot\|a\|_1+\alpha\cdot R_1(x)+\gamma\cdot R_2(h).$$

- Here, $R_1(x) = \sum\limits_{d\in D} 2^{1-o(d)} \sum_i |\Delta_i^d(x)|^p$, where $\Delta_i^d(x)$ is the difference operator, and
- $R_2(h) = \|\mathbf{C}h\|^2$, where $\mathbf{C}$ is the discrete Laplace operator.
- The ℓ^p-sum $\sum\limits_i |v_i(x)|^p$ is optimized as $\sum\limits_i \dfrac{(v_i(x^{(k)}))^2}{v_i^{2-p}}$, where $v_i = v_i(x^{(k-1)})$ for x from the previous iteration.
- This method results in the best blind image deconvolution.

7. Need for Improvement

- The current technique is based on minimizing the sum $|\Delta_x I|^p + |\Delta_y I|^p$.
- This is a discrete analog of the term $\left|\dfrac{\partial I}{\partial x}\right|^p + \left|\dfrac{\partial I}{\partial y}\right|^p$.
- For $p = 2$, this is the square of the length of the gradient vector and is, thus, rotation-invariant.
- However, for $p \neq 2$, the above expression is not rotation-invariant.
- Thus, even if it works for some image, it may not work well if we rotate this image.
- To improve the quality of image deconvolution, it is thus desirable to make the method rotation-invariant.
- We show that this indeed improves the quality of deconvolution.

8. Rotation-Invariant Modification: Description and Results

- We want to replace the expression $\left|\frac{\partial I}{\partial x}\right|^p + \left|\frac{\partial I}{\partial y}\right|^p$ with a rotation-invariant function of the gradient.
- The only rotation-invariant characteristic of a vector a is its length $\|a\| = \sqrt{\sum_i a_i^2}$.
- Thus, we replace the above expression with

$$\left(\left|\frac{\partial I}{\partial x}\right|^2 + \left|\frac{\partial I}{\partial y}\right|^2\right)^{p/2}.$$

- Its discrete analog is $((\Delta_x I)^2 + (\Delta_y I)^2)^{p/2}$.
- This modification leads to a statistically significant improvement in reconstruction accuracy $\|\widehat{x} - x\|_2$.

9. Testing the New Algorithm: Details

- To test the new method, we compared it with the original methods:
  - on the same "Cameraman" image use in the original method,
  - with the same values of the parameters ($\alpha = 1$, $\gamma = 5 \cdot 10^5$, $\tau = 0.125$, $\eta^1 = 1024$);
  - we applied the same Gaussian blurring with the variance of 5;
  - with the same S/N ratio corr. to $\sigma = 0.001$.
- We used the same criterion $\|x - \widehat{x}\|_2$ to gauge the deconvolution quality.
- Both methods start with randomly selected initial values $v_d^{1,1}$.
- Because of this, the results differ slightly when we re-apply the algorithm to the same image.

10. Testing the New Algorithm (cont-d)

- Because of the statistical character of the results:
  - we apply both algorithms to the same image several times, and
  - we use statistical criteria to decide which method is better.
- To perform this comparison, we applied each of the two algorithms 30 times.
- To make the results more robust, we eliminated the smallest and the largest value of this distance.
- The averages of the remaining 28 distances are:
  - for the original algorithm 1195.21,
  - for the new algorithm, $1191.01 < 1195.21$.

11. Testing the New Algorithm: Results

- To check whether this difference is statistically significance, we applied the t-test for two independent means:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\left(\frac{(N_1 - 1) \cdot s_1^2 + (N_2 - 1) \cdot s_2^2}{N_1 + N_2 - 2}\right) \cdot \left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}.$$

- The null hypothesis is that both samples comes from the populations with same mean.
- For the two above samples, computations lead to rejection with $p = 0.002$.
- This is much smaller than the p-values 0.01 and 0.05 normally used for rejecting the null hypothesis.
- Therefore, the *modified algorithm is statistically significantly better than the original one.*

12. Conclusions and Future Work

- Often, we need to reconstruct an image in situations when we do not know the blurring function.
- There exist empirically successful algorithms for such blind image deconvolution.
- While the current methods are reasonably efficient, they are not yet perfect; for example:
  - the current method correctly reconstructs the standard "Cameraman" image from its blurred version,
  - but when we rotated this image, the quality of the reconstruction drastically decreased.
- Making the first-order regularization terms rotation-invariant statistically significantly improves the image.
- It may be a good idea to try a similar replacement for second-order regularization terms.

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