Towards a Better Understanding of Space-Time Causality: Kolmogorov Complexity and Causality as a Matter of Degree

Vladik Kreinovich[1] **and Andres Ortiz**[2]
[1]Department of Computer Science
[2]Departments of Mathematical Sciences and Physics
University of Texas at El Paso, El Paso, TX 79968, USA
aortiz19@miners.utep.edu, vladik@utep.edu

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1. Defining Causality Is Important

- *Causal relation* $e \preccurlyeq e'$ between space-time events is one of the fundamental notions of physics.
- *In Newton's physics*, it was assumed that influences can propagate with an arbitrary speed:
$$e = (t, x) \preccurlyeq e' = (t', x') \Leftrightarrow t \le t'.$$
- In *Special Relativity*, the speeds of all the processes are limited by the speed of light c:
$$e = (t, x) \preccurlyeq e' = (t', x') \Leftrightarrow c \cdot (t' - t) \ge d(x, x').$$
- In the *General Relativity Theory*, the space-time is curved, so the causal relation $\preccurlyeq$ is even more complex.
- Different theories, in general, make different predictions about the causality $\preccurlyeq$.
- So, to experimentally verify fundamental physical theories, we need to experimentally check whether $e \preccurlyeq e'$.

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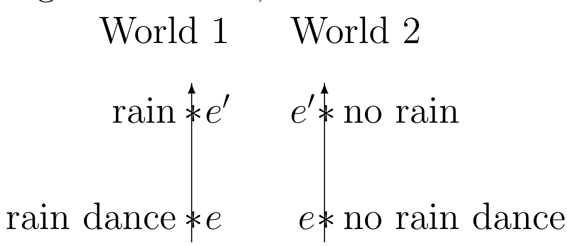
2. Defining Causality: Challenge

- Intuitively, $e \preccurlyeq e'$ means that:
  - what we do in the vicinity of e
  - changes what we observe at e'.
- If we have *two* (or more) copies of the Universe, then:
  - in one copy, we perform some action at e, and
  - we do not perform this action in the second copy.
- If the resulting states differ, this would indicate $e \preccurlyeq e'$:



- Alas, in reality, we only observe *one* Universe, in which we either perform the action or we do not.

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3. Algorithmic Randomness and Kolmogorov Complexity: A Brief Reminder

- If we flip a coin 1000 times and still get get all heads, common sense tells us that this coin is not fair.
- Similarly, if we repeatedly flip a fair coin, we cannot expect a periodic sequence $0101\ldots01$ (500 times).
- Traditional probability theory does not distinguish between random and non-random sequences.
- Kolmogorov, Solomonoff, Chaitin: a sequence $0\ldots0$ isn't random since it can be printed by a short program.
- In contrast, the shortest way to print a truly random sequence is to actually print it bit-by-bit: $\texttt{printf}(01\ldots)$.
- Let an integer $C>0$ be fixed. We say that a string x is *random* if $K(x)\ge \mathrm{len}(x)-C$, where

$$K(x)\stackrel{\text{def}}{=}\min\{\mathrm{len}(p):p \text{ generates } x\}.$$

4. The Corresponding Notion of Independence

- If y is independent on x, then knowing x does not help us generate y.
- If y depends on x, then knowing x helps compute y; example:
  - knowing the locations and velocities x of a mechanical system at time t
  - helps compute the locations and velocities y at time $t + \Delta t$.
- Let an integer $C > 0$ be fixed. We say that a string y is *independent* of x if $K(y \,|\, x) \geq K(y) - C$, where

$$K(y \,|\, x) \stackrel{\text{def}}{=} \min\{\text{len}(p) : p(x) \text{ generates } y\}.$$

- We say that a string y is *dependent* on the string x if

$$K(y \,|\, x) < K(y) - C.$$

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5. How to Define Space-Time Causality: First Seeming Reasonable Idea

- At first glance, we can check whether $e \preccurlyeq e'$ as follows:
  - First, we perform observations and measurements in the vicinity of the event e, and get the results x.
  - We also perform measurements and observations in the vicinity of the event e', and produce x'.
  - If x' depends on x, i.e., if $K(x'\,|\,x) \ll K(x')$, then we claim that e can casually influence e'.
- If $e \preccurlyeq e'$, then indeed knowing what happened at e can help us predict what is happening at e'.
- However, the inverse is not necessarily true.
- We may have $x \approx x'$ because both e and e' are influenced by the same past event e''.
- Example: both e and e' receive the same signal from e''.

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6. Towards a Working Definition of Causality

- According to modern physics, the Universe is quantum in nature; for microscopic measurements:
  - we cannot predict the exact measurement results,
  - we can only predict probabilities of different outcomes; the actual observations are truly random.
- For each space-time event e:
  - we can set up such a random-producing experiment in the small vicinity of e, and
  - generate a random sequence r_e.
- This random sequence r_e can affect *future* results.
- So, if we know the random sequence r_e, it may help us predict future observations.
- Thus, if $e \preccurlyeq e'$, then for some observations x' performed in the small vicinity of e', we have $K(x' \,|\, r_e) \ll K(x')$.

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7. Discussion and Resulting Definition

- *Reminder:* when $e \preccurlyeq e'$, then the random sequence r_e can affect the measurement results at e':

$$K(x' \,|\, r_e) \ll K(x').$$

- If $e \not\preccurlyeq e'$, then the random sequence r_e cannot affect the measurement results at e': $K(x' \,|\, r_e) \approx K(x')$.
- So, we arrive at the following semi-formal *definition:*
  - For a space-time event e, let r_e denote a random sequence generated in the small vicinity of e.
  - We say $e \preccurlyeq e'$ if for some observations x' performed in the small vicinity of e', we have

$$K(x' \,|\, r_e) \ll K(x').$$

- Our definition follows the ideas of casuality as *mark transmission*, with the random sequence as a mark.

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8. This Definition Is Consistent with Physical Intuition

- If $e \preccurlyeq e'$, then we can send all the bits of r_e from e to e'.
- The signal x' received in the vicinity of e' will thus be identical to r_e.
- Thus, generating x' based on r_e does not require any computations at all: $K(x' \,|\, r_e) = 0$.
- Since the sequence $x' = r_e$ is random, we have
$$K(x') \geq \mathrm{len}(x') - C.$$
- When $r_e = x'$ is sufficiently long ($\mathrm{len}(x') > 2C$), we have $K(x') \geq \mathrm{len}(x') - C > 2C - C = C$, hence
$$0 = K(x' \,|\, r_e) < K(x') - C \text{ and } K(x' \,|\, r_e) \ll K(x').$$
- So, our definition is indeed in accordance with the physical intuition.

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9. Randomness Is a Matter of Degree

- *Reminder*: a sequence x is random if $K(x) \geq \text{len}(x) - C$ for some $C > 0$.
- For a given sequence x, its *degree of randomness* $d(x)$ can be defined by the smallest integer C for which

$$K(x) \geq \text{len}(x) - C.$$

- One can check that this smallest integer is equal to the difference $d(x) = \text{len}(x) - K(x)$.
- For *random* sequences, the degree $d(x)$ is small.
- For sequences which are *not random*, the degree $d(x)$ is large.
- In general, the smaller the difference $d(x)$, the more random is the sequence x.

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10. Space-Time Causality Is a Matter of Degree

- Our definition of causality is that $K(x' \,|\, r_e) < K(x') - C$ for some large integer C.
- The larger the integer C, the more confident we are that an event e can causally influence e'.
- It is therefore reasonable to define a *degree of causality* $c(e, e')$ as the largest integer C for which
$$K(x' \,|\, r_e) < K(x') - C.$$
- One can check that this largest integer is equal to the difference $c(e, e') = K(x') - K(x' \,|\, r_e) - 1$.
- The larger this difference $c(e, e')$, the more confident we are that e can influence e'.
- In other words, just like randomness turns out to be a matter of degree, causality is also a matter of degree.

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11. Remaining Open Problems

- It is desirable to explore possible physical meaning of such "degrees of causality" $c(e, e')$.
- Maybe this function $c(e, e')$ is related to relativistic metric – the amount of proper time between e and e'?
- Another open problem: the above definition works for objects in a small vicinity of one spatial location.
- In quantum physics, not all objects are localized in space-time.
- We can have situations when the states of two spatially separated particles are entangled.
- It is desirable to extend our definition to such objects as well.

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12. Conclusions

- We propose a new operationalist definition of causality $e \preccurlyeq e'$ between space-time events e and e'.
- Namely, to check whether an event e can casually influence an event e', we:
  - generate a truly random sequence r_e in the small vicinity of the event e, and
  - perform observations in the small vicinity of the event e'.
- If some observation results x' (obtained near e') *depend* on r_e, then we claim that $e \preccurlyeq e'$.
- On the other hand, if all observation results x' are *independent* on r_e, then we claim that $e \not\preccurlyeq e'$.
- This new definition naturally leads to a conclusion that space-time causality is a matter of degree.

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