How Geophysicists' Intuition Helps Seismic Data Processing

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Expert Knowledge Is . . .
How Geophysical . . .
How De-Noising under . . .
Tikhonov . . .
Fuzzy-Motivated Solution
What If Signals Are . . .
Limitation of a . . .
Solution: Using Local . . .
Expert Knowledge: . . .

1. Expert Knowledge Is Needed In Geophysics

- In geophysics, signals come with noise; this noise affects the resulting images and maps.
- It is therefore desirable to minimize the effect of this noise.
- Sometimes, we know the probabilities of different values of signal and noise.
- In such situations, we can use statistical filtering techniques to find optimal de-noising.
- Since the original Wiener filter, many filtering statistical techniques have been invented.
- In geophysics, it is often impossible to directly measure the physical characteristics of the rocks at large depths.
- Thus, we do not know the actual probabilities.
- So, we have to rely on expert knowledge.

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2. How Geophysical Expert Knowledge Is Used Now

- Now, the geophysical expert knowledge is mainly used to select the most physically reasonable Earth model.
- This trial-and-error approach has led to many successful applications, such as finding oil.
- However, this search-in-the-dark processes is very time-consuming.
- We need faster ways to translate expert knowledge into data processing techniques.
- We show that fuzzy techniques can help with such translation.
- We show that the results are in good accordance with the empirically successful semi-heuristic methods.

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3. How De-Noising under Uncertainty Is Done Now: Case of Smooth Signals

- In many practical situations, we know that the actual signal $x(t)$ is smooth.
- In contrast, the observed signal $\widetilde{x}(t)$ contains non-smooth noise and is, thus, non-smooth.
- Often, we know how smooth is the actual signal: $\int (\dot{x}(t))^2\,dt \le b^2$ for some b.
- Among all such smooth signals, we find $X(t)$ which is the closest to $\widetilde{x}(t)$:

$$\int (X(t) - \widetilde{x}(t))^2\,dt \to \min\,.$$

- We want to minimize $\int (X(t) - \widetilde{x}(t))^2\,dt$ under the constraint $\int (\dot{x}(t))^2\,dt \le b^2$.
- According to Lagrange multiplier technique, this is equivalent to $\int (X(t) - \widetilde{x}(t))^2\,dt + \lambda \cdot \int (\dot{C}(t))^2\,dt \to \min\,.$

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4. Tikhonov Regularization: Problems

- In Fourier transform, the resulting *Tikhonov regularization* has the form $\hat{X}(\omega) = \dfrac{\hat{\tilde{x}}(\omega)}{1 + \lambda \cdot |\omega|^2}$.
- This works well when we know how smooth is the original signal, i.e., when we know λ.
- Often, we do not know smoothness.
- So, after applying this idea, we realize that an additional smoothing $\Delta\lambda$ is needed.
- Alas, applying this additional smoothing to $\hat{X}(\omega)$, we get a result different from smoothing w/$\lambda' = \lambda + \Delta\lambda$:

$$\frac{\hat{\tilde{x}}(\omega)}{(1 + \lambda \cdot |\omega|^2) \cdot (1 + \Delta\lambda \cdot |\omega|^2)} \neq \frac{\hat{\tilde{x}}(\omega)}{1 + \lambda' \cdot |\omega|^2}.$$

- Also, in geosciences, the signal is often discontinuous, since there are abrupt transitions (like Moho).

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5. Fuzzy-Motivated Solution

- We know that $X(t) \approx \widetilde{x}(t)$.
- Smoothness means that if t and t' are close, then $X(t) \approx X(t')$ and thus, $X(t) \approx \widetilde{x}(t')$.
- Let $\mu(t, t')$ describe closeness; then, for each t, we get $X(t) = \widetilde{x}(t')$ with degree $\mu(t, t')$.
- The usual centroid defuzzification leads to
$$X(t) = \frac{\int \mu(t, t') \cdot \widetilde{x}(t)\, dt'}{\int \mu(t, t')\, dt'}.$$
- Under reasonable assumptions, closeness is described by a Gaussian membership function
$$\mu(t, t') = \exp\left(-\frac{(t - t')^2}{2\sigma^2}\right).$$
- Thus, we get $X(t) = C \cdot \int \widetilde{x}(t') \cdot \exp\left(-\frac{(t - t')^2}{2\sigma^2}\right)\, dt'$.

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6. Fuzzy-Motivated Solution (cont-d)

- We get $X(t) = C \cdot \int \widetilde{x}(t') \cdot \exp\left(-\dfrac{(t-t')^2}{2\sigma^2}\right) dt'$.
- In Fourier transform, $\hat{X}(\omega) = \hat{\widetilde{x}}(\omega) \cdot \exp\left(\dfrac{1}{2} \cdot \sigma^2 \cdot \omega^2\right)$.
- In this case, two consecutive smoothings are equivalent to a single smoothing of this type:

$$\hat{X}(\omega) \cdot \exp\left(\frac{1}{2} \cdot (\sigma')^2 \cdot \omega^2\right) = \hat{\widetilde{x}}(\omega) \cdot \exp\left(\frac{1}{2} \cdot (\sigma'')^2 \cdot \omega^2\right),$$

$$\text{where } (\sigma'')^2 \stackrel{\text{def}}{=} \sigma^2 + (\sigma')^2.$$

- Moreover, one can prove that this is the only smoothing with this property.
- The resulting *shape regularization* has indeed been successfully used in geophysics (Sergey Fomel et al.).

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7. What If Signals Are Sometimes Not Smooth?

- Many geophysical structures have abrupt boundaries.
- Thus, the signals are not smooth: they have abrupt transitions corresponding to these boundaries.
- To describe such signals, it is thus natural to use piecewise smooth models (*splines*).
- Splines have indeed been efficiently used in data processing, in particular, in seismic data processing.
- However, splines do not explain what type of a transition this is.
- A more adequate description should take into account the specifics of the corresponding transitions.
- In general, specifics means that we have a family of models $m(t, \vec{c})$, with parameters $\vec{c} = (c_1, \ldots, c_k)$.

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8. Families of Models

- A smooth dependence can be locally well described by a linear model

$$x(t) \approx m(t) = c_1 + c_2 \cdot t.$$

- A wave can be locally described by a sinusoid

$$x(t) \approx m(t) = c_1 \cdot \cos(c_2 \cdot t + c_3).$$

- There are also finite-parametric models for such non-smooth phenomena as phase transitions.
- In all the above cases, a model with a finite number of parameters is only an approximation:
  - the longer the period of time that we need to cover,
  - the less accurate the model becomes.
- Thus, a reasonable idea is to use each such model only locally.

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9. Limitation of a Traditional Piece-Wise Smooth (Spline) Approach: Example

- A smooth function $x(t)$, in the vicinity of each point t_0, can be well approximated by a linear function:

$$x(t) \approx m(t) = x(t_0) + \dot{x}(t_0) \cdot (t - t_0).$$

- The further t from t_0, the less accurate the corresponding approximation.
- To make approximation accurate, we need to use different linear approximations on different time intervals:
  - first $m(t) = x(t_0) + \dot{x}(t_0) \cdot (t - t_0)$,
  - then $m(t) = x(t_1) + \dot{x}(t_1) \cdot (t - t_1)$, etc.
- At the border between two intervals, the derivative $\dot{m}(t)$ changes from $\dot{x}(t_0)$ to $\dot{x}(t_1)$.
- However, the original signal $x(t)$ was smooth!

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10. Solution: Using Local Attributes

- In the past, computers were limited, so we could only use a few local models.
- Nowadays, the computers are more powerful.
- It is now possible, for each moment t_0, to consider a model $x(t, \vec{c}(t_0))$ which is the best fit for $t \approx t_0$.
- This approach is consistent with the geophysicists' intuition; e.g., a geophysicist can meaningfully talk:
  - about a local frequency and amplitude of a wave,
  - and about how these quantities change as a wave changes.
- This approach has led to interesting practical results: e.g., to enhanced oil recovery.
- At present, geophysicists use *semi-heuristic* ideas to process the local models; let us use fuzzy instead.

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11. Expert Knowledge: Non-Smooth Case

- When a signal is not necessarily smooth, we cannot claim that the dependence $x(t)$ is smooth.
- However, it is still reasonable to claim that:
  - when we have local models $m(t, \vec{c}(t_0))$ and $m(t, \vec{c}(t_0'))$ corresponding to two nearby moments
    $$t_0' \approx t_0,$$
  - then the corresponding parameters $\vec{c}(t_0)$ and $\vec{c}(t_0')$ should be close.
- In other words, the dependence $\vec{c}(t_0)$ of the corresponding parameters on the time t_0 should be smooth.
- Thus, a natural idea is to apply the above-described expert-based smoothing to the dependence $\vec{c}(t_0)$.

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12. Expert Knowledge: Non-Smooth Case (cont-d)

- Let us apply the above-described expert-based smoothing to the dependence $\vec{c}(t_0)$ on t_0.
- To be more precise:
  - first, we determine, for each t_0, the values $\vec{c}(t_0)$ that best fit $\widetilde{x}(t)$ in the vicinity of t_0;
  - then, we apply the expert-based (Gaussian) smoothing to the resulting dependence $\vec{c}(t_0)$.
- This is, in effect, how the successful practical applications were obtained by Fomel et al.

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