# Why Smaller-Size Objects Affect the Flow Much More than Larger Ones: A Geometric Explanation with Applications Ranging from Volcanoes and Tornadoes to Blood, Fish, and Buildings Preservation

Laxman Bokati[1] and Vladik Kreinovich[1,2]
[1]Computational Science Program
[2]Department of Computer Science
University of Texas at El Paso, El Paso, Texas 79968, USA
lbokati@miners.utep.edu, vladik@utep.edu

## 1. Formulation of the problem

- What is the effect of objects of different size on a liquid or gaseous flow?
- At first glance, the larger the object, the more this object will affect the flow.
- However, in many different situations, empirical data shows the opposite effect: smaller-size objects have a much larger effect on the flow that larger-size ones.
- This phenomenon has been observed in real-life situations:
  - the flow of lava in a volcano,
  - the flow or air in a tornado,
  - the flow of blood in a body,
  - the fish-affected flow of water in the ocean, and
  - the destructive flow of seeped water in the old buildings.

## 2. Formulation of the problem (cont-d)

- This phenomenon occurs in different physical situations.
- This seems to indicate that there should be a general explanation for all such effects.
- There should be an explanation that does not depend on the specific physics.
- In this talk, we provide such a general explanation.
- Our explanation is based only on simple geometry and thus, does not depend on the underlying physics.

## 3. Volcanoes

- In volcanoes, hot lava – mixed with gases and rocks of different sizes – comes to the surface.
- In some volcanoes, lava come out as a continuous stream.
- In other volcanoes, lava accumulates and then gets released in a violent eruption.
- Violent eruptions often have catastrophic consequences.
- It is therefore important to be able to predict:
  - whether a volcano will erupt or
  - whether it will release its lava in a more "peaceful" way – as a continuous flow.
- Based on the general laws of hydrodynamics, researchers are able, in most cases, to predict whether an abrupt eruption is possible.

## 4. Volcanoes (cont-d)

- It should be mentioned that when eruption is possible:
  - the current models can only predict the very fact of the future eruption,
  - they cannot predict its exact time.
- These models assume, for simplicity, that the lava flow only contain the liquid lava and hot gases, they ignore the presence of rocks.
- In many cases, this simplification is well justified:
  - irrespective of how many rocks the actual lava flows contains,
  - these models usually provide good predictions.
- However, there have been quite a few puzzling cases, when:
  - the model predicted a smooth continuous lava flow, but
  - the volcano actually violently erupted.

## 5. Volcanoes (cont-d)

- A partial explanation of this puzzle came from a recent research that showed that:
  - in all the cases of unexpected explosions, lava had a significant portion of nano-size particles, while
  - in the cases of a smooth continuous lava flow, the overall amount of nano-size particles was much smaller.
- But this explanation leads to another puzzle:
- Why do smaller-size particles affect the properties of the flow while larger-size rocks do not have such an effect?

## 6. Tornadoes

- Tornadoes – rotating fast air flows – cause a lot of destruction.
- It is difficult to predict them.
- So maybe we can stop them – or at least slow them down and thus, make them less destructive – when they have already formed?
- At first glance, a natural idea is to inject dust into a tornado.
- This way, we increase the rotating mass, and thus, since the angular momentum remains constant, slow down the rotation.
- Surprisingly, in most cases, the corresponding experiments did not work: the tornado did not slow down.
- The only cases when the tornado did slow down somewhat was when the injected dust was fine-grained, consisting of micro-size particles.
- This result is puzzling: why do smaller-size particles affect the flow while larger-size particles do not have the same effect?

## 7. Blood

- Our bodies depend on blood.
- Blood circulates oxygen and nutrients to different parts of the body.
- Sometimes, blood becomes too viscous to circulate properly, and this causes serious problems.
- This happens, e.g., during many cases of so-called "long Covid", complications after a Covid infection.
- Interestingly, in such cases, what helps is *thrombocytapheresis* – partial removal of micro-size cells (called platelets) from the blood.
- Here, we face the same puzzling question.
- Why do smaller-size particles significantly affect the blood flow while the effect of larger-size particles is much smaller?

# 8. Fish

- Most biological creatures depend on oxygen.
- Thus, for deep-water creatures to survive, it is important to have a circulation of water between different depths.
- This way, oxygen consumed by these creatures will be replaced by oxygen coming from the surface.
- Similarly, the main source of energy is – eventually – the Sun.
- The sunlight practically does not penetrate deep water.
- So it is important that circulation between different depths bring nutrients to the deeper layers.
- In the ocean, there is a natural mixing of waters.
- However, this mixing is too slow to explain the richness of deep-water life.
- In addition to a natural mixing, there is also mixing caused by sea creatures.

## 9. Fish (cont-d)

- Many of the creatures move in all directions (including up and down), and thus, help the water circulate.
- At first glance, it seems like the larger the creature, the larger effect it has on water circulation.
- So, to check on this effect, researchers tried to measure circulation caused by large animals like whales.
- Surprisingly, they found out that the resulting mixing effect is too small to explain how oxygen and nutrients get into the depths.
- The results of these large-creature measurements were negative.
- So, researchers naturally assumed that:
  - the mixing effect caused by smaller creatures will be even smaller,
  - even if we take into consideration that fish may congregate in big shoals.
- So the researchers did not even bother to try to measure this effect.

## 10. Fish (cont-d)

- When they finally measured this effect, they got the second surprise:
  - that the mixing caused by small-size sea creatures is much larger than the mixing caused by larger creatures, and
  - that this small-size-creature-induced mixing can actually explain the current levels of oxygen and nutrient in the deep sea.
- Thus, here is another case of the same puzzling question:
- Why do smaller-size sea creatures have a large effect on water circulation that larger-size ones?

## 11. Old buildings

- With time, old buildings eventually deteriorate.
- One of the reasons for this is that water seeps into the walls, into the foundation, and causes damage.
- To preserve old buildings, to slow down their deterioration, it is important to prevent water from flowing.
- Many techniques have been invented for this purpose.
- One of the main problems with these techniques is that:
  - for historic buildings – buildings that we want to disrupt as little as possible,
  - the existing techniques are too invasive.
- Interestingly, a recent research shows that inserting nano-particles slows down the deterioration.

## 12. Old buildings (cont-d)

- It works even better than previous proposed techniques which were based on inserting larger-size water barriers.
- Again, the puzzling question is: why smaller-size particles have a larger effect on water flow than larger-size ones?

## 13. The ideal state of a flow: a geometric description

- According to statistical physics, all the processes, if undisturbed, eventually reach their most stable state.
- This state is characterized by the maximal entropy.
- Equations describing liquids and gases are invariant with respect to:
  - rotations,
  - shifts, and
  - scalings – i.e., dilations $\vec{x} \mapsto \lambda \cdot \vec{x}$.
- Thus, the most stable state of liquids and gases should also be invariant with respect to all these transformations.

## 14. The ideal state of a flow: a geometric description (cont-d)

- Invariance with respect to shifts means that the liquid or gas should be in the homogeneous state.
- All the characteristics of the liquid should be the same (at least locally) in all neighboring points.
- Invariance with respect to rotations imply that in the stable state, liquid or gas should be stationary; otherwise:
  - if they move we get a preferred direction – the direction in which the liquid or gas moves,
  - and thus, this state is not invariant with respect to rotations.

## 15. How perturbations change the state

- Eventually, the object will reach its final state.
- However, this process can take a long time; for example:
  - the Universe itself is already billions of years in the making, but
  - it has not yet (luckily for us) reached its final stable state.
- According to statistical physics, this change can be sped up by appropriate perturbations.
- For example, a small particle helps the overheated liquid to start boiling.
- If the perturbation has some invariances (= symmetries), its effect on the surrounding liquid also has the same invariances.
- Thus, it brings the surrounding flow closer to the state with these invariances.

## 16. How perturbations change the state (cont-d)

- From this viewpoint:
  - the more of the desired symmetries the perturbation has,
  - the closer it brings the surrounding flow to the desired fully invariant state.
- From this angle, let us compare the effect of smaller-size and larger-size particles.

## 17. Comparing symmetries of smaller-size and larger-size particles leads to the desired explanation

- By definition, a very-small-size particle is a particle whose size can be safely ignored.
- This is, in effect, what physicists call a point-wise particle.
- A point is invariant:
  - with respect to rotations around it, and
  - with respect to all scalings relative to this point.
- On the other hand, a larger-size body cannot be scale-invariant.
- Indeed, invariance with respect to a transformation means that:
  - if we have a point inside the body and apply the transformation to this point,
  - then we also get a point from this same body.
- However, by re-scaling points in a small vicinity of the body's central point, we can get all points in the 3D space.

## 18. Comparing symmetries of smaller-size and larger-size particles leads to the desired explanation (cont-d)

- Thus, we can get points which are so far away from the central point that they are no longer part of the body.
- At best, a body can be rotation-invariant: if its shape is close to a sphere.
- So, in all cases, a smaller-size particle has more desired symmetries that a larger-size one.
- Thus, smaller-size particles bring the state of the surrounding flow closer to the stable state (i.e., homogeneous and stationary state) than larger-size ones.

## 19. Why this is not a paradox

- This larger effect of smaller-size particles would be a paradox if we were taking about injecting energy into the flow.
- Of course, a smaller-size particle has much fewer energy than a larger-size one.
- So it cannot inject more energy into the flow than the larger-size particle.
- However, in our case, we are not talking about injecting energy.
- In all the above examples, the flow already has energy, and in examples like volcanoes or tornadoes, a huge amount of energy.
- What we are talking about is *not* processes that require injection of energy.
- Vice versa, we talk about processes like slowing down the flow, i.e., processes that decrease the flow's energy.

## 20. Why this is not a paradox (cont-d)

- For this purpose:
  - we do not need to inject energy,
  - we just need to trigger – and thus speed up – the process that, according to the 2nd Law of Thermodynamics, will happen anyway.
- Similarly, to make an overheated liquid start boiling, we do not need to inject any energy into it.
- All we need is a triggering particle, and this particle can be as small as possible.

## 21. How this explains all the above-described empirically observed effects

- The general idea is that adding smaller-size particles slows down the flow and makes it more homogeneous.
- For tornadoes, this means that the destructive airflow slows down, and thus, becomes less destructive.
- For volcanoes, it means that nano-particles prevent the lava from flowing.
- Thus, instead of flowing continuously, lava stays under the volcano.
- As more and more lava flows from lower levels into the same space, the pressure increases and eventually, an explosion happens.

## 22. How this explains all the above-described empirically observed effects (cont-d)

- Interestingly, in these two cases, the same physical effects leads, from the human viewpoint, to opposite consequences:
  - the presence of smaller-size particles make tornadoes less dangerous, but
  - a similar presence of smaller-size particles makes volcanoes much more dangerous.
- Similarly, we can explain all other phenomena described in the previous slides.
- For blood flows, the presence of smaller-size particles slows down the blood flow.
- So, we need to remove some of these particles to restore the healthy blood flow speed.

## 23. How this explains all the above-described empirically observed effects (cont-d)

- For fish in the ocean, smaller-size sea creatures bring the state of the sea closure to uniform.
- Thus, closer to the state in which the amounts of oxygen and nutrients are (approximately) the same at all depths.
- This is exactly what mixing is about.
- Finally, for historic buildings, smaller-size particles prevent water from seeping.
- This is exactly what is needed to slow down the building's deterioration.

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