What to Teach? How to Teach? How to Grade? Towards Mathematical Analysis of Teaching

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1. General Idea

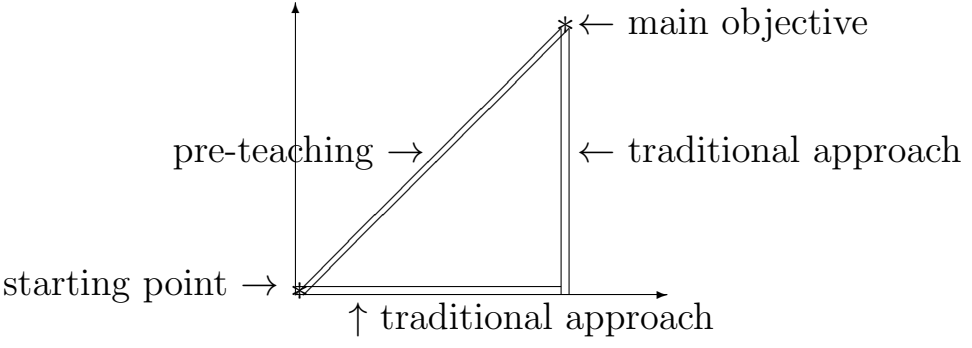
- Teaching is a very important human activity.
- Teaching is not easy.
- In contrast to many other activities, successful teaching strategies are often reasonably informal and thus, not easy to follow.
- So, a natural way to help instructors to adopt these strategies is to describe all aspects of teaching in more precise terms.
- Moving towards such more precise descriptions of various aspects of teaching is the main objective of this talk.
- Of course, we are still far from this ultimate objective.
- However, we hope that our preliminary results and ideas:
  - will be helpful to instructors, and
  - will inspire others to continue this endeavor and thus, bring us all close to the ideal of perfect teaching.

2. What to Teach: What We Cover

- In this talk, we provide a mathematical explanation for two aspects of this general question.
- First, we explain why pre-teaching is useful, when we provide basics of future topics before teaching them.
- Sophia Kovalevskaya saw math on wallpaper when a kid, and it helped her later.
- This is not only in math: the best way to study language is by immersion, not phonetics first.
- The best way to understand a research paper is to first read an abstract, etc.
- Second, we explain why teaching seemingly unrelated history etc. courses helps students be better in math.

3. Why Pre-Teaching: A Geometric Explanation

- We start at a point $(0, \ldots, 0)$ where a student has no knowledge in new topics.
- We want to reach a point $(100, \ldots, 100)$ when a student has perfect knowledge in all the topics.
- Traditionally, we first study one topic, going from $(0, \ldots, 0)$ to $(100, 0, \ldots, 0)$, then another topic, etc.
- Geometry shows that it is faster to use a straight line, i.e., to study all topics simultaneously whenever possible.



4. Why Language, History, Philosophy, etc.?

- In many application areas, we study problems for which:
  - once we have a candidate for a solution,
  - we can feasibly check whether it is indeed a solution.
- The class of all such problem is called NP. Examples:
  - a mathematician looking for a proof of a statement (or of its negation),
  - a physicist looking for a formula that fits all seemingly unrelated observations,
  - an engineer looking for a design that satisfies all given specifications.
- It is known that some problems from the class NP are the hardest: every problem from NP can be reduced to them.
- Such problems are known as NP-complete.

5. Why Language, History, Philosophy, etc. (cont-d)

- Important consequence:
  - once we have an efficient algorithm for solving several instances of an NP-complete problem,
  - it automatically helps solve instances of other NP problems.
- In particular, knowing how problems are solved in philosophy etc. can (and does) help solve mathematical problems.
- For the same reason, art and nature can help solve math problems.
- Also, math ideas can help solve problems in physics – which explains math's efficiency in physics noticed by E. Wigner.

6. How to Teach: Video or Text? Bullets or No Bullets?

- During the pandemic, many of us posted neatly typed lectures on the web, instead of the usual imperfect handwriting on a board.
- Surprisingly, some students missed the handwriting. Why?
- Our explanation:
  - Some people (left-brain in pop-psychology terms) better understand sequential information.
  - Others (right-brain ones) better understand pictures and diagrams.
- Ideal solution: accommodate both, present both the text *and* the pictures.
- Same with publications: publish both the text *and* the slides.

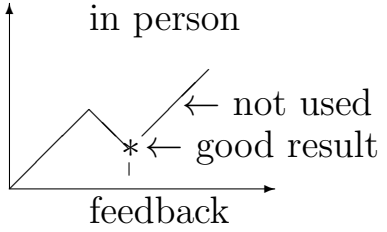
7. How to Teach: Why Online Teaching Amplifies the Differences Between Instructors

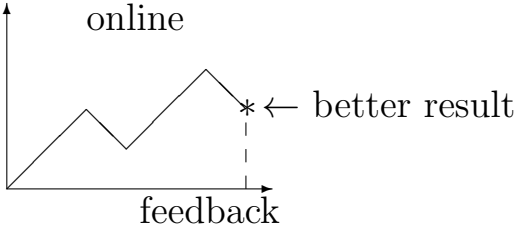
- Empirical studies show that online, good instructors teach even better, and not-so-experienced instructors teach even worse.
- Why worse can be naturally explained: in online teaching, feedback is limited.
- In person, an instructor can see that his/her techniques do not work, and stop using them before long.
- Online, it will take longer to realize this, so not-so-efficient techniques will be used longer.
- But why do good instructors teach better online?

8. How to Teach: Why Online Teaching Amplifies the Differences Between Instructors (cont-d)

- Our explanation is based on the same delayed feedback.
- The immediate efficiency of a technique depends on many factors: weather, students mood, etc.
- At some point, because of these other factors, teaching effectiveness decreases even when techniques are good.
- In person, the instructor immediately notices this decrease and thus, stops using the effective technique too early.
- Online, with limited feedback, the effective technique will be used longer – thus leading to more effective teaching.

9. How to Teach: Why Online Teaching Amplifies the Differences Between Instructors (cont-d)





10. How to Grade So As to Decrease Procrastination

- In general, a way to improve learning is to better motivate students.
- However, empirical data shows that there is an aspect not improved by motivation: procrastination.
- Many students start working on an assignment too late, turn them in late, and this affects their learning.
- A usual way to fight procrastination is to penalize students for late assignments.
- However, such a penalty is unfair for students who work hard but still struggle with the material.
- Proposed solution: explicit larger late penalties for students who do well and smaller ones for those who struggle.

11. How to React to Student Evaluations

- When most students comment that the course was too fast, a good idea is to slow down next time.
- Similarly, when most students comment that the course was too slow, next time we should speed up.
- But what if half of the students comments that the course was too fast and half that it was too slow?
- In this case, a frequent reaction is to think that the speed was just right and repeat the same speed next time.
- However, this risks failing half of the class – for which the course was too fast.
- These students need help, e.g., additional assignments to practice.
- Simply adding new assignments is usually not possible: instructors are already overbusy with grading.

12. How to React to Student Evaluations (cont-d)

- A natural solution: give fewer assignments to good students, and use this time to grade more assignments for students at risk.
- Specifically:
  - A small number of assignments is compulsory for all students.
  - Students who do not do well on one of these compulsory assignments get additional assignments.
- This way, hopefully, everyone will learn – without further burdening the instructor.

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