

Applications of 1-D Versions of Image Referencing Techniques to Hydrology and to Patient Rehabilitation

Roberto Araiza, Martine Ceberio,
Naga Suman Kanagala, Vladik Kreinovich
Department of Computer Science
University of Texas at El Paso
500 W. University
El Paso, TX 79968, USA
contact vladik@utep.edu

Gang Xiang
Philips Healthcare Informatics
El Paso, TX 79912

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1. Outline

- *Problems*: we consider two seemingly unrelated problems:
 - the hydrology problem of relation between groundwater and surface water, and
 - a problem of identification of human gait in neuro-rehabilitation.
- *Tools*: soft computing-motivated algorithms originally developed for image referencing.
- *Result*: these tools can be efficiently used in both problems.

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2. First Problem: Introduction

- *Fact:* most of our water consumption comes from ground-water reservoirs.
- *Problem:* contamination of surface water can lead to an eventual contamination of groundwater.
- *Good situations:*
 - often, it takes a long time for surface water to seep in;
 - during this time, many contaminants are safely dissolved.
- *Bad situations:* the surface water takes a very short time to propagate into the groundwater reservoirs.
- *Important:* determine a delay t_0 between
 - the change in the surface water and
 - the resulting change in the groundwater.

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3. Formulation of the Problem in Precise Terms

- *Objective:* determine a delay t_0 between
 - the change in the surface water and
 - the resulting change in the groundwater.
- *We measure:*
 - a characteristic $x(t)$ of the surface water at different moments of time t , and
 - a characteristic $x'(t)$ of ground water.
- *Idea:* the state of the groundwater is related to the state of the surface water at time $t - t_0$:

$$x'(t) \approx x(t - t_0).$$

- *Resulting formulation:* find the value t_0 for which

$$x'(t) \approx x(t - t_0).$$

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4. Second Problem: Introduction

- *Fact:* many neurological diseases drastically decrease the patient's ability to walk w/o physical assistance.
- *Examples:* stroke, traumatic body injury, and spinal cord injury
- *Extensive rehabilitation* is needed to re-establish normal gait.
- *At present:* rehabilitation requires gait assessment by highly qualified experienced clinicians.
- *Problem:* difficult to access, high costs.
- *It is desirable:* to automate gait assessment:
 - to make rehabilitations easier to access, and
 - to decrease the rehabilitation cost.

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5. Formulation of the Problem in Precise Terms

- A gait is measured by the dependence $x'(t)$ of some characteristic on time.
- *Example:* the acceleration or the angle between different parts of the foot.
- *The gait assessment* means comparing
 - the recorded patient's gait with
 - a standard (average) gait $x(t)$ of healthy people of the same age, body measurements, etc.
- *Problem:* patients walk slower.
- *Solution:* appropriately shift and “scale” the standard gait.
- *Resulting formulation:* find the values t_0 and λ for which

$$x'(t) \approx x(\lambda \cdot t - t_0).$$

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6. Straightforward Algorithm and Its Limitations

- *Given:* the patient gait $x'(t)$ and the standard gait $x(t)$.
- *Find:* the values t_0 and λ for which

$$x'(t) \approx x(\lambda \cdot t - t_0).$$

- *Straightforward idea:* try all possible shifts and scalings.
- *Limitations:* this is computationally very intensive.
- *Objective:* to design an efficient algorithm for finding the optimal combination of a shift and a scaling.
- *Our idea:* adjust the known image referencing techniques that use Fast Fourier Transform.

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7. Why Fourier-Based Methods

- *Simplest case:* find the shift t_0 for which $x'(t) \approx x(t - t_0)$.
- *Notation:* let n be the number of moments of time for which we know $x(t)$.
- *Natural formalization:* least squares method – find t_0 that minimizes $I \stackrel{\text{def}}{=} \int (x'(t) - x(t - t_0))^2 dt$.

- *Simplification:*

$$I = \int (x'(t))^2 dt + \int x(t - t_0)^2 dt - 2 \int x'(t) \cdot x(t - t_0) dt.$$

- *Analysis:* the first two terms do not depend on t_0 .
- *Conclusion:* find t_0 for which the convolution $J(t_0) \stackrel{\text{def}}{=} \int x'(t) \cdot x(t - t_0) dt$ is the largest.
- *Computation time:* we need n convolutions, with n steps each; overall time $O(n) \cdot O(n) = O(n^2)$.

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8. Why Fourier-Based Methods (cont-d)

- *Fact:* convolution is one of the main techniques in signal processing.
- *Fact:* we can compute convolution $J(t_0)$ faster:
 - first, we apply FFT to the original signals, resulting in functions $F(\omega)$ and $F'(\omega)$;
 - then, for each frequency ω , we compute the product

$$P(\omega) \stackrel{\text{def}}{=} F(\omega) \cdot (F')^*(\omega);$$

- third, we apply FFT^{-1} to the resulting function $P(\omega)$, and get the desired convolution $J(t_0)$.
- Finally, we find t_0 for which $J(t_0) \rightarrow \max$.
- FFT requires $O(n \cdot \log(n))$ steps, multiplication and search for t_0 is $O(n)$.
- So, we find t_0 in time $O(n \cdot \log(n)) + O(n) = O(n \cdot \log(n)) \ll O(n^2)$.

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9. Shift Detection: Resulting Algorithm

- *Ideal case:* $x'(t) = x(t - t_0)$, hence:
 - $F'(\omega) = e^{2\pi \cdot i \cdot (-\omega \cdot t_0)} \cdot F(\omega)$;
 - here, the ratio $R(\omega) \stackrel{\text{def}}{=} P(\omega)/|P(\omega)|$ is equal to $R(\omega) = e^{2\pi \cdot i \cdot (-\omega \cdot t_0)}$;
 - thus, the FFT^{-1} of $R(\omega)$ is equal to $I(t) = \delta(t + t_0)$;
 - so, t_0 is the only value for which $I(-t) \neq 0$.
- *In practice:* $x'(t) \approx x(t - t_0)$, so:
 - we apply FFT to the original signals $x(t)$, $x'(t)$ and compute their Fourier transforms $F(\omega)$ and $F'(\omega)$;
 - we compute the product $P(\omega) = F(\omega) \cdot (F')^*(\omega)$ and the ratio $R(\omega) = P(\omega)/|P(\omega)|$;
 - we apply FFT^{-1} to $R(\omega)$ and get $I(t)$;
 - we find t_0 for which $|I(-t_0)| \rightarrow \max$.

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10. General Case

- *General case:* $x'(t) \approx x(\lambda \cdot t - t_0)$.
- *Analysis:* the magnitudes $M(\omega) = |F(\omega)|$ and $M'(\omega) = |F'(\omega)|$ differ by scaling: $M'(\omega) \approx (1/\lambda) \cdot M(\omega/\lambda)$.
- *Idea:* in *log frequencies* $\rho = \log(\omega)$, scaling becomes shift-like: $\rho \rightarrow \rho - b$, where $b = \log(\lambda)$.
- *Resulting algorithm:*
 - transform $M(\omega)$ and $M'(\omega)$ to log frequencies;
 - use the above FFT-based algorithm to determine the corresponding shift $\log(\lambda)$;
 - from the corresponding “shift” value, reconstruct the scaling coefficient λ ;
 - re-scale $x(t)$ to $x(\lambda \cdot t)$ and use the same FFT-based algorithm to compute the shift t_0 .

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11. The Role of Soft Computing: A Comment

- *We need:* describe the degree s with which the signals $x'(t)$ and $x(t - t_0)$ are similar.
- *Expression used:* $d = \int (x'(t) - x(t - t_0))^2 dt$.
- *How we derived it:* made a heuristic transition.
- *Desirable:* have a better justification.
- *Idea:* fuzzy techniques have been specifically designed to describe terms like “similar”.
- *Implementation:* use a Gaussian membership function $\mu(x' - x) = \exp(-c \cdot (x' - x)^2)$ and $a \cdot b$ as “and”.
- *Conclusion:* $s = \prod_t \exp(-c \cdot (x'(t) - x(t - t_0))^2)$.
- *Result:* by taking logarithms, we see that

$$s \rightarrow \max \Leftrightarrow d \rightarrow \min .$$

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12. Conclusions

- Most of our water consumption comes from groundwater reservoirs.
- Contamination of surface water can lead to an eventual contamination of groundwater.
- If the surface water takes a long time to propagate, the contaminants will dissolve.
- On the other hand, if the propagation time is short, we may have contamination.
- It is thus important to estimate the propagation time.
- In this paper, we design a fast algorithm that uses Fast Fourier Transform to gauge this propagation time.

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13. Conclusions (cont-d)

- Many neurological diseases drastically decrease the patient's ability to walk without physical assistance.
- To re-establish normal gait, patients undergo extensive rehabilitation.
- At present, rehabilitation requires gait assessment by highly qualified experienced clinicians.
- To make rehabilitations easier to access, it is desirable to automate gait assessment.
- In this paper, we design a fast algorithm that uses Fast Fourier Transform for gait assessment.

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