Quantum Computation Techniques for Gauging Reliability of Interval and Fuzzy Data

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1. Outline

- Traditionally, we assume that the interval bounds are correct, and that (fuzzy) expert estimates are correct.
- In practice, measuring instruments and experts are not 100% reliable.
- Usually, we know the percentage of such outlier unreliable measurements.
- However, it is desirable to check that the reliability of the actual data is indeed within the given percentage.
- The problem of checking (gauging) this reliability is, in general, NP-hard.
- In reasonable cases, there exist feasible algorithms for solving this problem.
- We show that quantum computing can speed up the computation of reliability of given data.

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2. Two main sources of information

- *In practice:* we want to know the state of objects.
- *In science:* we are simply interested in this state.
- *Example:* we want to know the river's water level.
- *In engineering:* we need the information about the state of the world to change the situation.
- *Example:* how to build a dam to prevent flooding.
- *Most accurate reliable estimate of each quantity:* measurement.
- In many cases, it is too difficult or too expensive to measure all the quantities.
- In such situations, we can ask the experts to estimate the values of these quantities.
- Measurements and expert estimates are thus the two main sources of information about the real world.

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3. Measurement uncertainty and interval data

- The result $\widetilde{x}$ of a measurement is usually somewhat different from the actual (unknown) value x.
- Usually, the manufacturer of the measuring instrument (MI) gives us a bound Δ on the measurement error:

$$|\Delta x| \le \Delta, \text{ where } \Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$$

- Once we know the measurement result $\widetilde{x}$, we can conclude that the actual value x is in $[\widetilde{x} - \Delta, \widetilde{x} + \Delta]$.
- In some situations, we also know the probabilities of different values $\Delta x \in [-\Delta, \Delta]$.
- In this case, we can use statistical techniques.
- However, often, we do not know these probabilities; we only know that x is in the interval $\mathbf{x} \stackrel{\text{def}}{=} [\widetilde{x} - \Delta, \widetilde{x} + \Delta]$.
- So, we need to process this interval data.

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4. Expert estimates and fuzzy data

- There is no guarantee of expert's accuracy.
- We can only provide bounds which are valid with some degree of certainty.
- This degree of certainty is usually described by a number from the interval $[0, 1]$.
- So, for each $\beta \in [0, 1]$, we have an interval $\mathbf{x}(\alpha)$ containing the actual value x with certainty $\alpha = 1 - \beta$.
- The larger certainty we want, the broader should the corresponding interval be.
- So, we get a nested family of intervals corresponding to different values α.
- *Alternative:* for each x, describe the largest α for which x is in $\mathbf{x}(\alpha)$; this α_{largest} is a membership function $\mu(x)$.

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5. Reliability of interval data

- *Usual assumption:* all measuring instruments (MI) functioned correctly.
- *Conclusion:* the resulting intervals $[\widetilde{x} - \Delta, \widetilde{x} + \Delta]$ contain the actual value x.
- *In practice:* a MI can malfunction, producing way-off values (outliers).
- *Problem:* outliers can ruin data processing.
- *Example:* average temperature in El Paso
  - based on measurements, $\dfrac{95 + 100 + 105}{3} = 100.$
  - with outlier, $\dfrac{95 + 100 + 105 + \underline{\mathbf{0}}}{4} = 75.$
- *Natural idea:* describe the probability p of outliers.
- *Solution:* out of n results, dismiss $k \stackrel{\text{def}}{=} p \cdot n$ largest values and k smallest.

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6. Need to gauge the reliability of interval data

- *Ideal case:* all measurements of the same quantity are correct.
- *Fact:* resulting intervals $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}$ contain the same (actual) value x.
- *Conclusion:* $\bigcap\limits_{i=1}^{n} \mathbf{x}^{(i)} \neq \emptyset$.
- *Reality:* we have outliers far from x, so $\bigcap\limits_{i=1}^{n} \mathbf{x}^{(i)} = \emptyset$.
- *Expectation:* out of n given intervals, $\geq n-k$ are correct – and hence have a non-empty intersection.
- *Conclusion:*
  - to check whether our estimate p for reliability is correct,
  - we must check whether out of n given intervals, $n-k$ have a non-empty intersection.

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7. Need to gauge reliability of interval data: multi-D case

- In practice, a measuring instrument often measure several different quantities $x_1, \ldots, x_d$.
- Due to uncertainty, after the measurement, for each quantity x_i, we have an interval $\mathbf{x}_i$ of possible values.
- So, the set of all possible values of the tuple $x = (x_1, \ldots, x_d)$ is a *box*

  $$X = \mathbf{x}_1 \times \ldots \times \mathbf{x}_d = \{(x_1, \ldots, x_d) : x_1 \in \mathbf{x}_1, \ldots, x_d \in \mathbf{x}_d\}.$$

- Thus:
  - to check whether our estimate p for reliability is correct,
  - we must be able to check whether out of n given boxes, $n - k$ have a non-empty intersection.

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8. How to gauge reliability of fuzzy data

- *Fact:* experts are sometimes wrong, so their estimates are way off.
- *Idea:* to gauge the reliability of the experts by the probability p that an expert is wrong.
- *Example:* $p = 0.1$ means that we expect 90% of the experts to provide us with correct bounds $X(0)$.
- *Comment:* we may have different probabilities p for different certainty levels α.
- *Conclusion:*
  - to check whether the data fits given reliability estimates,
  - we must therefore be able to check whether out of n given boxes, $n-k$ have a non-empty intersection.

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9. Resulting Computational problem: box intersection problem

Thus, both in the interval and in the fuzzy cases, we need to solve the following computational problem:

- Given:
  - integers d, n, and k; and
  - n d-dimensional boxes
$$X^{(j)} = [\underline{x}_1^{(j)}, \overline{x}_1^{(j)}] \times \ldots \times [\underline{x}_n^{(j)}, \overline{x}_n^{(j)}],$$
    $j = 1, \ldots, n$, with rational bounds $\underline{x}_i^{(j)}$ and $\overline{x}_i^{(j)}$.
- *Check* whether
  - we can select $n - k$ of these n boxes
  - in such a way that the selected boxes have a non-empty intersection.

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10. Results

- *First result:* in general, the above computational problem is NP-hard.
- *Meaning:* no algorithm is possible that solves all particular cases of this problem in reasonable time.
- *In practice:* the number of d of quantities measured by a sensor is small: e.g.,
  - a GPS sensor measures 3 spatial coordinates;
  - a weather sensor measures (at most) 5:
    - temperature,
    - atmospheric pressure, and
    - the 3 dimensions of the wind vector.
- *Second result:* for a fixed dimension d, we can solve the above problem in polynomial time $O(n^d)$.

11. Algorithm: description and need for speed up

- *Lemma:* if a set of boxes has a common point, then there is another common vector whose all components are endpoints.
- *Proof:* move to an endpoint in each direction.
- *Number of endpoints:* n intervals have $\le 2n$ endpoints.
- *Bounds on computation time:* we have $\le (2n)^d$ combinations of endpoints, i.e., polynomial time.
- *Remaining problem:* n^d is too slow;
  - for $n = 100$ and $d = 5$, we need 10^{10} computational steps – very long but doable;
  - for $n = 10^4$ and $d = 5$, we need 10^{20} computational steps – which is unrealistic.

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12. Use of quantum computing

- *Idea:* use Grover's algorithm for quantum search.
- *Problem:* search for a desired element in an unsorted list of size N.
- *Without using quantum effects:* we need – in the worst case – at least N computational steps.
- *A quantum computing algorithm* can find this element much faster – in $O(\sqrt{N})$ time.
- *Our case:* we must search $N = O(n^d)$ endpoint vectors.
- *Quantum speedup:* we need time $\sqrt{N} = O(n^{d/2})$.
- *Example:* for of $n = 10^4$ and $d = 5$,
  - the non-quantum algorithm requires a currently impossible amount of 10^{20} computational steps,
  - while the quantum algorithm requires only a reasonable amount of 10^{10} steps.

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13. Conclusion

- In traditional interval computations, we assume that
  - the interval data corresponds to guaranteed interval bounds, and
  - that fuzzy estimates provided by experts are correct.
- In practice, measuring instruments are not 100% reliable, and experts are not 100% reliable.
- We may have estimates which are "way off", intervals which do not contain the actual values at all.
- Usually, we know the percentage of such outlier unreliable measurements.
- It is desirable to check that the reliability of the actual data is indeed within the given percentage.

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In this paper, we have shown that:

- in general, the problem of checking (gauging) this reliability is computationally intractable (NP-hard);
- in the reasonable case
  - when each sensor measures a small number of different quantities,
  - it is possible to solve this problem in polynomial time;
- quantum computations can drastically reduce the required computation time.

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