Assessment of Functional Impairment in Human Locomotion: Fuzzy-Motivated Approach

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1. Formulation of the Problem

- Neurological disorders – e.g., the effects of a stroke – affect human locomotion (such as walking).
- In most cases, the effect of a neurological disorder can be mitigated by applying an appropriate rehabilitation.
- For the rehabilitation to be effective, it is necessary to be able:
  - to correctly diagnose the problem,
  - to assess its severity, and
  - to monitor the effect of rehabilitation.
- At present, this is mainly done subjectively, by experts who observe the patient.
- This is OK for the diagnosis, but for rehabilitation, a specialist can see a patient only so often.

2. Formulation of the Problem (cont-d)

- It is desirable to *automatically* gauge how well the patient progresses.
- To measure the gait $x(t)$, we can use:
  - inertial sensors that measure the absolute and relative location of different parts of the body, and
  - electromyograph (EMG) sensors that measure the electric muscle activity during the motion.
- By comparing $x(t)$ with gait of healthy people and with previous patient's gait, we can:
  - gauge how severe is the gait disorder, and
  - gauge whether the rehabilitation is helping.
- *Problem:* signals $x(t)$ corresponding to patients and to healthy people are similar.

3. Need for Fuzzy Techniques

- Specialists *can* distinguish between signals corr. to patients and healthy people.
- We want to *automate* this specialists' skill.
- Specialists describe their decisions by using imprecise ("*fuzzy*") words from natural language.
- Formalizing such words is one of the main tasks for which *fuzzy techniques* have been invented.
- Fuzzy techniques have been used to design efficient *semi-heuristic* assessment systems.
- The objective of this paper is to provide a *theoretical justification* for the existing fuzzy systems.
- The existence of such a justification makes the results of the system *more reliable*.

4. Pre-Processing of Gait Signal

- Motions differ by speed: the same person can walk slower or faster.
- To reduce the effect of different speeds, we re-scale time $x'(T) = x(t_0 + T \cdot T_0)$, where
  - t_0 is the beginning of the gait cycle,
  - T_0 is the gain cycle, and
  - the new variable T describe the position of the sensor reading on the gait cycle.
- For example:
  - the value $x'(0)$ describes the sensor's reading at the beginning of the gait cycle,
  - the value $x'(0.5)$ describes the sensor's reading in the middle of the gait cycle,
  - the value $x'(0.25)$ describes the sensor's reading at the quarter of the gait cycle.

5. Pre-Processing of Gait Signal (cont-d)

- Motions also differ by intensity.
- To reduce the effect of different intensities, we re-scale the signal $x(t)$ so that:
  - the smallest value on each cycle is 0, and
  - the largest value on each cycle is 1.
- Such a scaling has the form $X(T) = \dfrac{x'(T) - \underline{x}}{\overline{x} - \underline{x}}$, where:
  - $\underline{x}$ is the smallest possible value of the signal $x'(T)$ during the cycle, and
  - $\overline{x}$ is the largest possible value during the cycle.
- After re-scaling, all we have to do is compare:
  - the (re-scaled) observed signal $X(T)$ with
  - a similarly re-scaled signal $X_0(T)$ corresponding to the average of normal behaviors.

6. Fuzzy Gait Assessment System

- An expert describes the gait by specifying how the motion looked like at different (p) parts of the gait cycle.
- For each part, we form a triangular membership function $\mu(x)$ that best describes the corr. values $X(T)$.
- We want the support (a, b) of $\mu(x)$ to be narrow *and* to contain many observed values x_i.
- Pedrycz's approach: find parameters a, b, m for which
$$\frac{\sum_{i=1}^{n} \mu(x_i)}{b - a} \to \max .$$
- The gait on each part is described by three parameters (a, b, m), so overall we need $N = v3p$ parameters.
- A patient's gait is described by $g_1, \ldots, g_N \in [0, 1]$.
- The normal gait is described by $n_1, \ldots, n_N \in [0, 1]$.

7. Fuzzy Gait Assessment System (cont-d) and Our Result

- A sequence $g_1, \ldots, g_N$ can be viewed as a fuzzy set g.
- A sequence $n_1, \ldots, n_N$ can be viewed as a fuzzy set n.
- So, we can define degree of similarity between patient's gait and normal gait as

$$s = \frac{|g \cap n|}{|g \cup n|} = \frac{\sum\limits_{i=1}^{N} \min(g_i, n_i)}{\sum\limits_{i=1}^{N} \min(g_i, n_i)}.$$

- Our result: when the number of parts p is large enough, we have

$$s \approx 1 - \frac{1}{C} \cdot \int |x(t) - x_0(t)|\, dt.$$

- Thus, the larger the integral, the more severe the disorder.

8. Explanation of the Reformulated Formula

- Let's explain why $\int |\Delta x(t)|\,dt$, where $\Delta x(t) \stackrel{\text{def}}{=} x(t) - x_0(t)$, is a good measure of the disorder's severity.
- The effect is different for different behaviors.
- It is reasonable to gauge the severity of a disorder by the worst-case effect of this difference.
- For each objective, the effectiveness E of this activity depends on the differences $\Delta x(t_i)$.
- The differences $\Delta x(t_i)$ are small, so we can linearize the dependence: $\Delta E = \sum c_i \cdot \Delta x(t_i)$.
- There is a bound M on possible values of $|c_i|$.
- The largest value of $\sum c_i \cdot \Delta x(t_i)$ under the constraint $|c_i| \le M$ is equal to $M \cdot \sum |\Delta x(t_i)|$.
- Thus, the worst-case effect is indeed proportional to $\sum |\Delta x(t_i)|$, i.e., to $\int |\Delta x(t)|\,dt$.

9. Conclusion and Future Work

- Many traumas and illnesses result in motion disorders.
- In many cases, the effects of these disorders can be decreased by an appropriate rehabilitation.
- Different patients react differently to the current rehabilitation techniques.
- To select an appropriate technique, it is therefore extremely important to be able to gauge:
  - how severe is the current disorder and
  - how much progress has been made in the process of rehabilitation.
- At present, this is mostly done subjectively, by a medical doctor periodically observing the patient's motion.
- When a certain therapy does not help, the doctor can change the rehabilitation procedure.

10. Conclusion and Future Work (cont-d)

- It is desirable to make frequent evaluations, to make sure that the procedure indeed improves the patient.
- For that, it is desirable to come up with ways to automatically access the patient's progress.
- In previous papers, fuzzy techniques were used to design semi-heuristic assessment techniques.
- In this paper, we provide a theoretical justification for these techniques.
- In the future, it is desirable:
  - to enhance these fuzzy-based assessment techniques
  - by combining them with fuzzy-based techniques for modeling gait (and other motions).

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12. Proof of the Main Result

- On each part i, the motion changes slightly from the midpoint $x(t_i)$: all observed values $x(t)$ are $x(t) \approx x(t_i)$.
- Hence, the values a, b, and m are also close to $x(t_i)$, so

$$s = \frac{3 \cdot \sum_{i=1}^{n} \min(x(t_i), x_0(t_i))}{3 \cdot \sum_{i=1}^{n} \max(x(t_i), x_0(t_i))} = \frac{\sum_{i=1}^{n} \min(x(t_i), x_0(t_i))}{\sum_{i=1}^{n} \max(x(t_i), x_0(t_i))}.$$

- Since $\Delta x(t) = x(t) - x_0(t)$, we get $x(t) = x_0(t) + \Delta x(t)$, and:

$$s = \frac{\sum_{i=1}^{n} \min(x_0(t_i) + \Delta x(t_i), x_0(t_i))}{\sum_{i=1}^{n} \max(x_0(t_i) + \Delta x(t_i), x_0(t_i))}.$$

13. Proof (cont-d)

- Reminder: $s = \dfrac{\sum\limits_{i=1}^{n} \min(x_0(t_i) + \Delta x(t_i), x_0(t_i))}{\sum\limits_{i=1}^{n} \max(x_0(t_i) + \Delta x(t_i), x_0(t_i))}$.
- Here, if $\Delta x(t_i) \geq 0$, then

$$\min(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i).$$

- If $\Delta x(t_i) < 0$, then

$$\min(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i) + \Delta x(t_i).$$

- Similar formulas hold for max, so for $s_0 \stackrel{\text{def}}{=} \sum\limits_{i=1}^{n} x_0(t_i)$, we get

$$s = \frac{s_0 + \sum\limits_{i:\Delta x(t_i)<0} \Delta x(t_i)}{s_0 + \sum\limits_{i:\Delta x(t_i)\geq 0} \Delta x(t_i)}.$$

14. Proof (cont-d)

- Dividing both the numerator and the denominator by s_0, we conclude that

$$s = \frac{s_0 + \sum\limits_{i:\Delta x(t_i)<0} \Delta x(t_i)}{s_0 + \sum\limits_{i:\Delta x(t_i)\ge 0} \Delta x(t_i)} = \frac{1 + \sum\limits_{i:\Delta x(t_i)<0} \dfrac{\Delta x(t_i)}{s_0}}{1 + \sum\limits_{i:\Delta x(t_i)\ge 0} \dfrac{\Delta x(t_i)}{s_0}}.$$

- Since $|\Delta x(t_i)| \ll x(t_i)$, we can use the fact that

$$\frac{1+a}{1+b} \approx (1+a)\cdot(1-b+\ldots) = 1+a-b+\ldots$$

- Thus, $s \approx 1 + \sum\limits_{i:\Delta x(t_i)<0} \dfrac{\Delta x(t_i)}{s_0} - \sum\limits_{i:\Delta x(t_i)\ge 0} \dfrac{\Delta x_0(t_i)}{s_0}$.

- Hence $s = 1 + \dfrac{1}{s_0} \cdot \left(\sum\limits_{i:\Delta x(t_i)<0} \Delta x(t_i) - \sum\limits_{i:\Delta x(t_i)\ge 0} \Delta x(t_i) \right)$.

15. Proof (final part)

- Reminder: $s = 1 + \frac{1}{s_0} \cdot \left(\sum\limits_{i:\Delta x(t_i)<0} \Delta x(t_i) - \sum\limits_{i:\Delta x(t_i)\ge 0} \Delta x(t_i) \right)$.
- So, $s \approx 1 - \frac{1}{s_0} \cdot \sum\limits_{i=1}^{n} |\Delta x(t_i)|$.
- Once we multiply this sum by $\Delta t = t_{i+1} - t_i$, we get an integral sum $\sum\limits_{i=1}^{n} |\Delta x(t_i)| \cdot \Delta t$ for the interval

$$\int |\Delta x(t)|\, dt.$$

- So, the dissimilarity (i.e., the severity of the disorder) is proportional to the integral $I \stackrel{\text{def}}{=} \int |\Delta x(t)|\, dt$.