Semi-Heuristic Target-Based Fuzzy Decision Procedures: Towards a New Interval Justification

Christian Servin[1], Van-Nam Huynh[2], and Yoshiteru Nakamori[2]

[1]Computational Science Program, University of Texas at El Paso
El Paso, TX 79968, christians@miners.utep.edu
[2]Japan Advanced Institute of Science and Technology (JAIST)
1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan
{huynh, nakamori}@jaist.ac.jp

1. Traditional Approach to Decision Making: Reminder

- The quality of each possible alternative is characterized by the values of several quantities.
- For example, when we buy a car, we are interested in its cost, its energy efficiency, its power, size, etc.
- For each of these quantities, we usually have some desirable range of values.
- Often, there are several different alternatives all of which satisfy all these requirements.
- The traditional approach assumes that there is an objective function that describes the user's preferences.
- We then select an alternative with the largest possible value of this objective function.

2. Traditional Approach to Decision Making: Limitations

- The traditional approach to decision making assumes:
  - that the user knows exactly what he or she wants — i.e., knows the objective function – and
  - that the user also knows exactly what he or she will get as a result of each possible decision.
- In practice, the user is often uncertain:
  - the user is often uncertain about his or her own preferences, and
  - the user is often uncertain about possible consequences of different decisions.
- It is therefore desirable to take this uncertainty into account when we describe decision making.

3. Fuzzy Target Approach (Huynh-Nakamori)

- For each numerical characteristic of a possible decision, we form two fuzzy sets:
  - $\mu_i(x)$ describing the users' ideal value;
  - $\mu_a(x)$ describing the users' impression of the actual value.
- For example, a person wants a well done steak, and the steak comes out as medium well done.
- In this case, $\mu_i(x)$ corresponds to "well done", and $\mu_a(x)$ to "medium well done".
- The simplest "and"-operation (t-norm) is $\min(a, b)$; so, the degree to which x is both actual *and* desired is

$$\min(\mu_a(x), \mu_i(x)).$$

- The degree to which there exists x which is both possible and desired is $d = \max\limits_x \min(\mu_a(x), \mu_i(x))$.

4. Detailed Derivation of the d-Formula

- We know:
  - a fuzzy set $\mu_i(x)$ describing the users' ideal value;
  - the fuzzy set $\mu_a(x)$ describing the users' impression of the actual value.
- For crisp sets, the solution is possibly satisfactory if some of the possibly actual values is also desired.
- In the fuzzy case, we can only talk about the degree to which the proposed solution can be desired.
- A possible decision is satisfactory if either:
  - the actual value is x_1, and this value is desired,
  - or the actual value is x_2, and this value is desired,
  - $\ldots$
- Here x_1, x_2, $\ldots$, go over all possible values of the desired quantity.

5. Derivation of the d-Formula (cont-d)

- For each value x_k, we know:
  - the degree $\mu_a(x_k)$ with which this value is actual, and
  - the degree $\mu_i(x_k)$ to which this value is desired.
- Let us use $\min(a, b)$ to describe "and" – the simplest possible choice of an "and"-operation.
- Then we can estimate the degree to which the value x_k is both actual *and* desired as

$$\min(\mu_a(x_k), \mu_i(x_k)).$$

- Let us use $\max(a, b)$ to describe "or" – the simplest possible choice of an "or"-operation.
- Then, we can estimate the degree d to which the two fuzzy sets match as

$$d = \max_x \min(\mu_a(x), \mu_i(x)).$$

6. Fuzzy Target Approach: How Are Membership Functions Elicited?

- In many applications (e.g., in fuzzy control), the shape of a membership function does not affect the result.
- Thus, it is reasonable to use the simplest possible membership functions – symmetric triangular ones.
- To describe a symmetric triangular function, it is sufficient to know the support $[\underline{x}, \overline{x}]$ of this function.
- So, e.g., to get the membership function $\mu_i(x)$ describing the desired situation:
  - we ask the user for all the values $a_1, \ldots, a_n$ which, in their opinion, satisfy the requirement;
  - we then take the smallest of these values as $\underline{a}$ and the largest of these values as $\overline{a}$;
  - finally, we form symmetric triangular $\mu_i(x)$ whose support is $[\underline{a}, \overline{a}]$.

7. Fuzzy Target Approach: Successes and Remaining Problems

- The above approach works well in many applications.
- *Example:* it predicts how customers select a handcrafted souvenir when their ideal ones is not available.
- *Problem:* this approach is heuristic, it is based on selecting:
  - the simplest possible membership function and
  - the simplest possible "and"- and "or"-operations.
- Interestingly, we get *better* predictions than with more complex membership functions and "and"-operations.
- In this paper, we provide a justification for the above semi-heuristic target-based fuzzy decision procedure.

8. Analyzing the Problem

- *Reminder:* all we elicit from the experts is two intervals:
  - an interval $[\underline{a}, \overline{a}] = [\widetilde{a} - \Delta_a, \widetilde{a} + \Delta_a]$ describing the set of all *desired* values, and
  - an interval $[\underline{b}, \overline{b}] = [\widetilde{b} - \Delta_b, \widetilde{b} + \Delta_b]$ describing the set of all the values which are *possible*.
- Based on these intervals, we build triangular membership functions $\mu_i(x)$ and $\mu_a(x)$ centered in $\widetilde{a}$ and $\widetilde{b}$.
- For these membership functions,

$$d = \max_x \min(\mu_a(x), \mu_i(x)) = 1 - \frac{|\widetilde{b} - \widetilde{a}|}{\Delta_a + \Delta_b}.$$

- This is the formula that we need to justify.

9. Our Main Idea

- If we knew the exact values of a and b, then we would conclude $a = b$, $a < b$, or $b < a$.
- In reality, we know the values a and b with uncertainty.
- Even if the actual values a and b are the same, we may get approximate values which are different.
- It is reasonable to assume that if the actual values are the same, then $\text{Prob}(a > b) = \text{Prob}(b > a)$, i.e.,
$$\text{Prob}(a > b) = 1/2.$$
- If the probabilities that $a > b$ and that $a < b$ differ, this is an indication that the actual value differ.
- Thus, it's reasonable to use $|\text{Prob}(a > b) - \text{Prob}(b > a)|$ as the degree to which a and b may be different.

10. How To Estimate $\text{Prob}(a > b)$ and $\text{Prob}(b > a)$

- If we knew the exact values of a and b, then we could check $a > b$ by comparing $r \stackrel{\text{def}}{=} a - b$ with 0.
- In real life, we only know a and b with interval uncertainty, i.e., we only know that
$$a \in [\widetilde{a} - \Delta_a, \widetilde{a} + \Delta_a] \text{ and } b \in [\widetilde{b} - \Delta_b, \widetilde{b} + \Delta_b].$$
- In this case, we only know the range $\mathbf{r}$ of possible values of $r = a - b$; interval arithmetic leads to
$$\mathbf{r} = [(\widetilde{a} - \widetilde{b}) - (\Delta_a + \Delta_b), (\widetilde{a} - \widetilde{b}) + (\Delta_a + \Delta_b)].$$
- We do not have any reason to assume that some values from $\mathbf{r}$ are more probable and some are less probable.
- It is thus reasonable to assume that all the values from $\mathbf{r}$ are equally probable, i.e., r is *uniformly* distributed.
- This argument is widely used in data processing; it is called *Laplace Principle of Indifference.*

11. How To Estimate Probabilities (cont-d)

- We estimate $\text{Prob}(a > b)$ as $\text{Prob}(a - b > 0)$.
- We estimate $\text{Prob}(a < b)$ as $\text{Prob}(a - b < 0)$.
- We assumed that $r = a - b$ is uniformly distributed on
$$[(\widetilde{a} - \widetilde{b}) - (\Delta_a + \Delta_b), (\widetilde{a} - \widetilde{b}) + (\Delta_a + \Delta_b)].$$
- We can compute $\text{Prob}(a - b > 0)$, $\text{Prob}(a - b < 0)$, and
$$|\text{Prob}(a > b) - \text{Prob}(b > a)| = \frac{|\widetilde{a} - \widetilde{b}|}{\Delta_a + \Delta_b}.$$
- Since $d = 1 - \dfrac{|\widetilde{b} - \widetilde{a}|}{\Delta_a + \Delta_b}$, we get
$$d = 1 - |\text{Prob}(a > b) - \text{Prob}(b > a)|.$$
- We have produced a new justification for the d-formula.
- This justification that does not use any simplifying assumptions about memb. f-s and/or "and"-operations.