Interval and Symmetry Approaches to Uncertainty – Pioneered by Wiener – Help Explain Seemingly Irrational Human Behavior: A Case Study

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1. Compromise Effect

- A customer shopping for an item has choices: some cheaper, some more expensive but of higher quality.
- Examples: shopping for a camera, for a hotel room.
- Researchers asked the customers to select one of the three randomly selected alternatives.
- They expected all three to be selected with equal probability.
- Instead, in the overwhelming majority of cases, customers selected the intermediate alternative.
- The intermediate alternative provides a compromise between the quality and cost.
- So, this phenomenon was named *compromise effect*.

2. Why This Is Irrational?

- Selecting the middle alternative seems reasonable.
- But let's consider alternatives $a_1 < a_2 < a_3 < a_4$ sorted by price (and quality).
- If we present the user with three choices $a_1 < a_2 < a_3$, the user will select the middle choice a_2.
- This means that, to the user, a_2 is better than a_3.
- But if we present the user with three other choices $a_2 < a_3 < a_4$, the same user will select a_3.
- So, to the user, the alternative a_3 is better than a_2.
- If in a pair-wise comparison, a_3 is better, then the first choice is wrong, else the second choice is wrong.
- In both cases, one of the two choices is irrational.

3. This is Not Just an Experimental Curiosity, Customers' Have Been Manipulated This Way

- At first glance, this seems like an optical illusion or a logical paradox: interesting but not very important.
- Actually, it is important: customers have been manipulated into buying a more expensive product.
- If there are two types of a product, a company adds an even more expensive third option.
- Recent research shows the compromise effect only happens when a customer has no additional information.
- In situations when customers were given access to additional information, their selections were consistent.
- However, in situation when decisions need to be made under major uncertainty, this effect is clearly present.
- How to explain such a seemingly irrational behavior?

4. Wiener's Symmetry Approach: Main Idea

- Main idea:
  - if the situation is invariant with respect to some natural symmetries,
  - then it is reasonable to select an action which is also invariant with respect to all these symmetries.
- This approach has indeed been helpful in dealing with uncertainty. In particular, it explains:
  - the use of a sigmoid activation function $s(z) = \dfrac{1}{1+\exp(-z)}$ in neural networks,
  - the use of the most efficient t-norms and t-conorms in fuzzy logic,
  - etc.

5. What Do We Know About the Utility of Each Alternative?

- The utility of each alternatives comes from two factors:
  - the first factor u_1 comes from the quality: the higher the quality, the better – i.e., the larger u_1;
  - the second factor u_2 comes from price: the lower the price, the better – i.e., the larger u_2.
- We have alternatives $a < a' < a''$ characterized by pairs $u(a) = (u_1, u_2)$, $u(a') = (u'_1, u'_2)$, and $u(a'') = (u''_1, u''_2)$.
- We do not know the utility values, we only know that

$$u_1 < u'_1 < u''_1 \text{ and } u''_2 < u'_2 < u_2.$$

- Since we only know the order, we can mark the values u_i as L (Low), M (Medium), and H (High).
- Then $u(a) = (L, H)$, $u(a') = (M, M)$, $u(a'') = (H, L)$.

6. Natural Transformations and Symmetries

- We do not know a priori which of the utility components is more important.
- It is thus reasonable to treat both components equally.
- So, swapping the two components is a reasonable transformation:
  - if we are selecting an alternative based on the pairs

    $$u(a) = (L, H),\ \ u(a') = (M, M),\ \text{and}\ u(a'') = (H, L),$$

  - then we should select the exact same alternative based on the "swapped" pairs

    $$u(a) = (H, L),\ \ u(a') = (M, M),\ \text{and}\ u(a'') = (L, H).$$

7. Transformations and Symmetries (cont-d)

- Similarly, there is no reason to a priori prefer one alternative versus the other.
- So, any permutation of the three alternatives is a reasonable transformation.
- We start with

$$u(a) = (L, H), \quad u(a') = (M, M), \quad u(a'') = (H, L).$$

- If we rename a and a'', we get

$$u(a) = (H, L), \quad u(a') = (M, M), \quad u(a'') = (L, H).$$

- For example:
  - if we originally select an alternative a with

$$u(a) = (L, H),$$

  - then, after the swap, we should select the same alternative – which is now denoted by a''.

8. What Can We Conclude From These Symmetries

- We start with
$$u(a) = (L, H), \quad u(a') = (M, M), \quad u(a'') = (H, L).$$
- If we swap u_1 and u_2, we get
$$u(a) = (H, L), \quad u(a') = (M, M), \quad u(a'') = (L, H).$$
- Now, if we also rename a and a'', we get
$$u(a) = (L, H), \quad u(a') = (M, M), \quad u(a'') = (H, L).$$
- These are the same utility values with which we started.
- So, if originally, we select a with $u(a) = (L, H)$, in the new arrangements we should also select a.
- But the new a is the old a''.
- So, if we selected a, we should select a'' – a contradiction.

9. What Can We Conclude (cont-d)

- We start with

$$u(a) = (L, H), \quad u(a') = (M, M), \quad u(a'') = (H, L).$$

- If we swap u_1 and u_2, we get

$$u(a) = (H, L), \quad u(a') = (M, M), \quad u(a'') = (L, H).$$

- Now, if we also rename a and a'', we get

$$u(a) = (L, H), \quad u(a') = (M, M), \quad u(a'') = (H, L).$$

- These are the same utility values with which we started.
- So, if originally, we select a'' with $u(a'') = (H, L)$, in the new arrangements we should also select a.
- But the new a'' is the old a.
- So, if we selected a'', we should select a – a contradiction.

10. Summarizing

- We start with

$$u(a) = (L, H), \quad u(a') = (M, M), \quad u(a'') = (H, L).$$

- If we swap u_1 and u_2, we get

$$u(a) = (H, L), \quad u(a') = (M, M), \quad u(a'') = (L, H).$$

- Now, if we also rename a and a'', we get

$$u(a) = (L, H), \quad u(a') = (M, M), \quad u(a'') = (H, L).$$

- We cannot select a – this leads to a contradiction.
- We cannot select a'' – this leads to a contradiction.
- The only consistent choice is to select a'.
- This is exactly the compromise effect.

11. Conclusion

- Experiments show that:
  - when people are presented with three choices $a < a' < a''$ of increasing price and increasing quality,
  - and they do not have detailed information about these choices,
  - then in the overwhelming majority of cases, they select the intermediate alternative a'.
- This "compromise effect" is, at first glance, irrational:
  - selecting a' means that, to the user, a' is better than a'', but
  - in a situation when the user is presented with $a' < a'' < a'''$, the user prefers a'' to a'.
- We show that a natural symmetry approach explains this seemingly irrational behavior.

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