Why Ideas First Appear in Informal Form? Why It Is Very Difficult to Know Yourself? Fuzzy-Based Explanation

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1. Ideas first appear in informal form

- From a purely mathematical viewpoint, it may seem that progress in physics and in other disciplines appears as a stroke of genius.

- Suddenly, someone comes up with equations.

- These equations fit the experiments perfectly, Nobel prize and fame follows – and the cycle continues with the next genius equation.

- Newton came up with equations of mechanics,

- Maxwell came up with equations of electrodynamics,

- Schroedinger came up with equations of quantum mechanics,

- Einstein came up with equations of General Relativity that describe gravity, etc.

- However, physicists know better.

- They know that equations did not come out of nowhere.

2. Ideas first appear in informal form (cont-d)

- In most cases:
  - first, there is an informal vague idea,
  - then this idea is getting more and more precise,
  - until finally someone comes up with absolutely precise equations.

- Newton based his mechanics equations on ideas of Galileo and others that have been analyzing mechanical motion for centuries.

- Schroedinger formalized ideas of de Broglie and others on the wave-particle duality.

- Einstein had vague ideas – that gravity disappears in a falling elevator and is, thus, largely equivalent to changing a coordinate system.

- It took him 10 years (and many unsuccessful attempts) to translate into precise equations that would fit with experimental data.

3. But why?

- At first glance, modern physics was formed explicitly as precise science, where:
  - everything is described by precise equations,
  - as opposed to vague ancient and medieval thinking about the physical world.

- Physicists strive to be as precise as possible, their objective is never to form vague ideas.

- So why do vague ideas form such an important role in physics – that they are practically a ubiquitous step in all scientific discoveries?

- In this paper, we provide a possible explanation for this phenomenon, an explanation using the ideas of fuzzy techniques.

4.   But why (cont-d)

- The main ideas behind this explanation enable us to explain other somewhat mysterious phenomena as well.

- E.g., the well-known fact that:
  - while a person knows much more about him/herself than about others,
  - analyzing oneself is much more difficult than analyzing others.

5. What physicists do

- Let us describe what physicists do in general terms.

- We have a set of observations – in many cases, a very large set of observations.

- We want to find equations that describe this set and that therefore will enable us to predict similar future phenomena.

6.   Usually, this is not easy

- Sometimes, these equations are reasonably easy to find.

- For example:
  - when Ohm simultaneously measured how changing the voltage V changes the current I flowing through a fixed conductor,
  - he plotted his results and saw that this dependence is linear $I = \text{const} \cdot V$: the famous Ohm's law.

- However, such cases are rare.

- In most real-life situations, there is no easy way to directly deduce the equations from the measurement results.

- In other words, there is a big distance between the experimental results and the equations.

7. So how do physicists solve their problems: a general description

- If you read some popular books on history of physics, you may get a false impression that evolution of physics is a straight line.

- Each new idea bring us closer and closer to the correct equations.

- But this is a false impression.

- Einstein formulated many wrong theories until he came up with the right equations.

- And many other physicists formulated even more wrong theories.

- From this viewpoint, if one takes all these failed attempts into account, the progress of physics looks:
  - not so much like a straight line
  - but rather as a random walk on each stage,
  - a random walk that after many attempts, reaches the desired objective (of fitting all the experimental data).

8. So how do physicists solve their problems: a more detailed description

- The problem with the random walk idea is that it means, in effect, that:

    - if we want to go from the initial state (experimental data) to the desired state (equations explaining this data),

    - then we have to consider almost all possible models within this distance from the desired state.

- As we have mentioned, in most real-life situations, the distance R between the initial state and the desired state is large.

- In such situations, there are many possible models in the corresponding vicinity of the initial state.

- The number of these models is proportional to the volume $\sim R^d$ of this vicinity (where d is the dimension of this space).

- This number is, therefore, large.

- Thus, it is not realistic to try all possible models in this vicinity.

- And this is not how models actually appeared.

- It is not that Schroedinger tried all possible differential equations of a given complexity – that would have taken forever.

- So how do physicists operate?

- Random walk works well when the distance R between the starting state and the desired state is small.

- In this case, we have not that many possible models within a vicinity of radius R, and it is indeed possible to try all of them.

- So, a natural idea is to somehow come up with intermediate objectives.

- Instead of looking for a precise model that explains all the data, it is desirable to come up with several intermediate objectives, so that:
    - the first desired state is close to the data,
    - the second desired state is close to the first desired state, etc., and
    - at the end, the last intermediate state is close to the desired final state.

- This is similar to crossing a mountain stream:
    - if this stream is wide, it is very difficult to cross, but
    - if we have rocks showing out of water, we can often easily go from one rock to another until we reach the other bank.

- So how can we form such intermediate states?

11. How to form intermediate states: a natural idea

- The ultimate objective is to have a model that fits all the data exactly:
  - i.e., that is consistent with all the available data,
  - of course, if we take into account that all available data comes from measurement uncertainty.

- Coming up with a model that exactly describes all the data often requires that we determine the values of a large number of parameters.

- Let us denote this number by p.

- In general, the number of possible models – i.e., the number of possible combinations of these parameter values – grows with p as N^p.

- Here N is the number of possible values of each parameter.

- This number grows exponentially with p, so for large p, it becomes not feasible to try all of them.

- Thus, as an intermediate step, we need to select a class of models that is characterized by fewer parameters.

- Of course, if we need a large number of parameters to fit the data *exactly*, there is no way to fit the data exactly with fewer parameters.

- So, on the intermediate stages, we need to use to use models that fit the data *approximately*.

- This naturally leads to all kind of ways to describe what the word "approximate" means.

- This word has some formalizations, but in general, it is an informal word.

- Thus, a natural general way to describe this imprecise word in precise terms is to use techniques for translating imprecise words:

  - from natural language
  - into precise language.

- This is exactly what fuzzy techniques are about.

13.   Why fuzzy, why not any other type of uncertainty?

- OK, we need to fit the data approximately, with some uncertainty.

- However, there are many different types of uncertainty: interval, probabilistic, etc., so why fuzzy?

- To answer this question, let us recall how we check whether a given formula is a good fit for the data.

- We can apply some statistical criterion.

- It will be probably enough to have the resulting paper published, but will we be convinced?

- Not always.

- For example, the usual $p < 0.05$ criterion means than in 5% of the cases, the observed match will be accidental.

- Such accidental matches – which cannot be reproduced in the follow-up experiments – have indeed been ubiquitous.

- To convince everyone that the formula fits the data we need to *see* that the data points are indeed close to the corresponding curve.

- Sometimes this closeness means that all the data points are within a certain bound away from the curve.

- Then, we have, in mathematical terms, an interval uncertainty.

- But this is not always the case.

- In many other cases, some deviations may be huge, but "on average", the data points are close to the curve.

- So, we have probabilistic uncertainty.

- In general, if we limit ourselves to any precise definition of uncertainty, we thus inevitably exclude many important cases when:

  - intuitively, the data fits,
  - but not in the sense of this particular definition.

- The only way to capture the general intuitive idea of fitness is:
  - to avoid precise formulations and
  - use imprecise ("fuzzy") terms – terms of which a natural language provides plenty.

- We humans, the product of billions of years of improving evolution, use such imprecise terms.

- This is an indication that using such terms is indeed more beneficial for our activities than using more precise alternatives.

- OK, so we use such imprecise terms intuitively.

- If we want to optimize enhance our knowledge acquisition process, we need to able to translate these terms into something more precise.

- And this is exactly what fuzzy technique was designed for (and for what it is still actively used):

  - to translate knowledge described by imprecise natural-language words

  - into a more precise computer-understandable numerical description.

- Thus, of all types of uncertainty, the best to use for this purpose is indeed fuzzy uncertainty.

17.   How ideas evolve – from this viewpoint

- Let us go back to our main argument:

  - once we found a model that provides an approximate description of the corresponding phenomena,

  - a natural next objective is to try to modify this approximate model into a more accurate one, etc.

- The whole idea of imprecision (fuzziness) is related to the fact that we are dealing with an approximation.

- The more accurate the model, the smaller the need for approximation.

- Thus, the smaller the need to take fuzziness into account.

- Thus, ideas that initially appear in a very fuzzy form become more and more precise, less and less fuzzy.

- Eventually, someone makes the last step and comes up with precise equations.

18. Unexpected conclusion

- The above analysis leads to a somewhat unexpected conclusion: that measurement errors are good for cognition.

- At first glance, this sounds counter-intuitive:

  - the more accurate the measurement, the more information we gain about the world,

  - so the easier it is to cognise it.

- However, this idea starts making sense if we take into account that at first, we want to come up with an approximate model.

19.   Historical examples

- Ohm's law is, in many cases, very accurate.

- In such cases, the voltage V is indeed proportional to the current I,

- In reality, the dependence of V on I is not linear:

  – if Ohm was able to perform very accurate measurements,

  – he would never be able to find a complex formula that exactly describes the empirical dependence.

- His measurements were not very accurate: there was a significant measurement error.

- So, linear dependence provided – within this accuracy – a very good fit for all the measurement results.

20. Historical examples (cont-d)

- Similarly,

  - if Newton had access to all modern super-precise astronomical measurements,

  - he would never be able to come up with his theory of gravitation.

- The exact description of this data requires much more complex equations of General Relativity.

- However, in Newton's days, measurements were not that accurate.

- So, he was able to come up with equations that – within his-time accuracy – fit all the observations perfectly.

21. We can avoid the curse of too accurate measurements, but this requires creativity

- Of course, this does not mean that if we have very accurate measurements, we cannot come up with theories.

- It just means that in this case, we need to come up with a reasonable approximate description.

- We need to know what needs to be taken into account and what details can be ignored.

- This is not easy – and this is an important part of the art of being a physicist.

- The great 20 century mathematician David Hilbert came up with the same equations of General Relativity as Einstein.

- Hilbert's paper was submitted only two weeks later than Einstein's.

- However, the big difference was that:
  - while Hilbert simply presented the same complex system of non-linear differential equations,
  - Einstein also provided approximate solutions and thus, the possibility to experimentally test this theory.

- It is not that Einstein was a better mathematician knowing some trick of solving non-linear differential equations that Hilbert did not.

- The reason why Einstein was more successful is that he knew which terms in these equations can be safely ignored.

- This made computations much easier.

23. Why Is It Difficult to Know Oneself

- Another consequence of the above general idea is that it explains a seemingly counterintuitive fact that:
  - even for people who are capable of providing great insights into the souls of others,
  - who can predict the others' behavior, who can explain the other's behavior and who can help others change their behavior if needed,
  - it is much more difficult to understand one's own soul, one's motivations and behaviors.

- For example, it is known that:
  - psychoanalysts – even the most advanced ones who can penetrate deeply into the patients' subconscious activity,
  - need help from others to understand their own behavior and their own problems.

24.   Why Is It Difficult to Know Oneself (cont-d)

- At first glance, it should be the other way around:
  - while we may know a lot about others,
  - we definitely know much more about ourselves.
- So, based on all this abundant information, we should be able to get a better description of ourselves than of others.
- However, the above argument shows that the reality is the opposite.
- When we know a few facts about another person, it is easier to fit these facts into a pattern.
- In contrast, when since we have an abundance of facts about ourselves, it is very difficult to come up with a description that fits all these facts.

25. How Can All This Be Used

- So, we have provided a qualitative explanation of the empirical phenomena.

- This may be intellectually satisfying, but from the pragmatic practical viewpoint, is this explanation useful?

- We cannot yet claim that we have used this explanation in practice, but we can envision how it can be used.

- In the past, data was scarce.

- Nowadays, in many application areas, we have an abundance of information, an abundance of data.

- Engineers and scientists are actively working on the problem on how to store and process all this data.

- Often, there is so much data that it its difficult to come up with models describing all this data.

26.   How Can All This Be Used (cont-d)

- This is a problem, e.g., with climate change: none of the existing models provides a perfect fit for all the observations.

- Our argument shows that:
  - this is not a reason to conclude – as some philosophers have been doing for centuries
  - that many processes in the world are not cognizable.

- If we cannot find a perfect fit, we should go for an approximate fit – e.g., fuzzify the data and try to fit it with fuzzy rules.

- This, by the way, is exactly what has made fuzzy control so successful.

- We should not desperately look for models that fit with $p < 0.05$ (or whatever criteria we use):
  - if we get such models, great,
  - but if not, we should start with approximate models and improve them until we finally (hopefully) get the accurate equations.

- This is what climatologists are doing, ignoring all criticisms from statisticians.

- This is what we should recommend to all the sciences which are trying to find an explanation for all the available new data.

28. Acknowledgments