

From Discrete (Crisp) to Continuous (Fuzzy) to a Combination of Discrete and Continuous: Case of Partial Credit

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1. Traditional discrete (crisp) way of grading in elementary school

- When we were studying mathematics in elementary school, there was no such thing as partial credit, grading was very discrete (crisp):
 - either your answer is correct, then you get a credit for this particular problem,
 - or your answer is wrong, then you do not get any credit for this problem.
- There was even a joke explaining this approach.
- A little big-city kid comes home and happily reports to his parents that he got a good grade for biology.
- When the teacher asked how many legs a chicken has, he said: “Three”.
- “But a chicken has only has to legs” – wondered his parents.
- “Yes”, the kid replied, “but everyone else said four.”

2. Partial credit – at higher educational levels

- Later on, in high school and at the university level, usually:
 - when the answer is wrong but the student performed several solution steps correctly,
 - this student no longer gets 0 point, he/she gets a partial credit.
- In other words, at this educational level, grading is done by using a continuous scale.
- The grade can go from 0 all the way to 1 (or to 10, or to 100) depending on how close the solution strategy is to the correct one.
- In effect, this is a fuzzy scale.
- Usually, unless some solution stages are more important, all these stages are valued the same.
- Thus, a student who correctly performed m out of n solution stages, gets partial credit m/n .

3. There is a problem with the usual way of assigning partial credit

- Here is a realistic (and sometimes real) problem with the usual approach to partial credit.
- Suppose that students take a test that consist of 10 equally important and equally complex problems.
- Each problem is worth 10 points, so overall, a student can get up to 100 points.
- As usual:
 - students who get 90 or more points get an A (“excellent”),
 - students who get between 80 and 89 points get a B (“good”),
 - students who get between 70 and 79 get a C (“satisfactory”),
 - students who get between 60 and 69 get a D, and
 - students who get smaller than 60 points get a failing grade F for this test.

4. There is a problem with the usual way of assigning partial credit (cont-d)

- Suppose that in each of these ten problem, the student has to follow 10 steps to reach a solution.
- Since each problem is worth 10 points, each step is worth 1 point.
- Suppose that in each of the ten problems, the student made one small mistake, i.e., performed one of the ten steps incorrectly.
- Then, according to the above-described usual arrangement, this student will get 9 out of 10 points for each of the ten problems.
- So, this student's overall grade for the test will be 90/100, and this students gets an A for this test.
- It may sound reasonable and fair, but let us look at it from the viewpoint of correct answers.
- All 10 answers are wrong, and still a student gets an A grade – indicating excellence.

5. There is a problem with the usual way of assigning partial credit (cont-d)

- Something is not right here.
- If this was happening in real life, and a person who needed to perform 10 computations gets all 10 wrong:
 - this person will be probably fired,
 - and he/she will definitely *not* be called an excellent worker.

6. This is a general problem of continuous grading

- One may think that this problem can be corrected by assigning different numbers of points.
- Yes, changing the grading scheme may help to eliminate the problem in this particular case.
- However, one can see that a similar problem appears in any continuous grading scheme:
 - if we assign the A letter grade to any numerical grade larger than or equal to some threshold $g_0 < 1$,
 - then a minor mistake in each of N problems – that makes it worth g_0/N points – will lead to an overall A,
 - while all N answers will still be wrong.
- In this talk, we show how to avoid such a situation.

7. Our main idea

- We have shown that:
 - a fully continuous grading scheme – where a grade for each problem can, in principle, take any value between 0 and 1
 - leads to an undesirable situation.
- So, a natural idea is to add some discreteness to grading.
- Specifically, the problem is caused by the fact that grades for an incorrect (but “almost” correct) answer can be close to 1.
- So, a natural idea is to only provide full grade of 1 when the solution is correct.
- And when even a small mistake is made, assign a value smaller than or equal to some smaller threshold $t_0 < 1$.

8. Our main idea (cont-d)

- With this arrangement, we get a grading scheme that combines discrete and continuous.
- Namely, possible grades for each problem can be:
 - either any number from 0 to t_0 – which corresponds to the continuous part,
 - or the value 1 – which corresponds to the discrete part.

9. But is this fair?

- At first glance, this may not sound fair to a student – a tiny mistake immediately drops his/her grade from 1 to t_0 .
- However, as we have mentioned, it makes perfect sense in real life.
- For example, when a construction company promises to build a building by a certain date, there is often a bonus for doing it on time.
- This bonus will *not* be paid if the construction is only a few days late.
- Maybe this is a way to make it sound fair to students:
 - instead of calling it a penalty,
 - call the difference $1 - t_0$ a *bonus* for the correct answer.

10. Comment

- Such a bonus-type description of grading is not unheard of.
- For example, when one of us (VK) was a student in St. Petersburg University, we had oral final exams.
- A student would randomly select a card with three questions.
- If the student answers all three questions correctly, he/she gets a guaranteed B (to be more precise, a grade of 4, a Russian equivalent of B).
- Of course, if the student fails to answer some of these three questions, the grade is lower.
- To raise the B grade to an A, the student has to answer all additional questions that the instructors ask.

11. Comment (cont-d)

- If the student does not answer these additional questions, the B grade remains.
- This way, B is not viewed as a penalty. Instead, A is viewed as a bonus for perfect knowledge.

12. What threshold should we choose

- We do not want to give a very low grade to a student whose knowledge of the material is almost perfect.
- On the other hand, if we make the threshold too close to 1:
 - students will not notice the difference and will not strive for having correct answers,
 - which is the main point of the proposed change in the grading policy.
- So, it is reasonable to select a threshold t_0 for which the difference $1 - t_0$ is the smallest still noticeable by an average student.
- To find such a difference, we can use the well known seven-plus-minus two law.
- According to this law, people usually classify objects into 7 ± 2 categories – between 5 and 9, on average 7.

13. What threshold should we choose (cont-d)

- From this viewpoint, for an average person, the smallest noticeable difference is $1/7$ – which is between 14 and 15%.
- So, it makes sense to select $1 - t_0 = 0.15$, i.e., $t_0 = 0.85$.
- Thus, we arrive at the following recommendation.

14. Resulting recommendation

- For a problem worth 10 points, we should give all 10 points only when the resulting answer is absolutely correct.
- If there is at least a minor mistake:
 - then immediately the grade goes down to 8.5 points
 - minus whatever points should be deducted for the student's mistakes.
- This grading strategy is equivalent to:
 - making each problem worth 8.5 points.
 - with the promise of additional 1.5 bonus points for an absolutely correct answer.
- This will avoid undesirable situations like we have described above:
 - when a student gets a A for the test in which all answers are incorrect,
 - almost correct but still incorrect.

15. Resulting recommendation (cont-d)

- Indeed, in this case, when all the answers are wrong, in the new grading system, the student will get at most 85 points for the test.
- This corresponds to B, not to A.

16. Additional problem

- An additional problem with the usual grading scheme is related to students' forgetting.
- Ideally, the overall grade for the class should reflect the level of the student's knowledge at the end of the class – and further in the future.
- However, in reality, students forget.
- As a result of this forgetting, some students who got a perfect A grade on all three midterm exams perform much worse on the final exam.
- Sometimes they even perform on a C level.
- However, in situations when the final exam is worth only 30 points:
 - losing 25/100 points on the final exam – by getting 75 on the final exam
 - simply means losing $0.3 \cdot 25 = 7.5$ points towards the class grade.
- So, when all the previous grades were perfect As, the student still gets 92.5 points for the class and thus, an A grade for the class.

17. Additional problem (cont-d)

- This is not just a hypothetical situation, it actually happened with one of our students.
- This is not right.
- By the end of the class, the student only shows a satisfactory level of knowledge.
- However, by giving this student an A grade we falsely certify that this student's level of knowledge is excellent.
- How can we avoid this problem?

18. Our proposal

- To avoid this problem, we need to make sure that the student's grade for the class cannot be significantly higher than this student's grade on the final exam.
- Indeed, the grade on the final exam reflects the student's level of knowledge at the end of the course.
- As we have argued, the significant difference means at least 14 points.
- So, the proposal is as follows:
 - if grade on the final exam is at least 15 points lower than the overall grade computed the usual way,
 - then the grade for the final exam become an official grade for the class.
- In the above case, the difference $92.5 - 75 = 17.5$ is larger than 15.
- So, the student would get, for the class, not an A, but a C – the letter grade corresponding to this student's 75 points on the final exam.

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