

How to Make Fuzzy Estimates Less Subjective: Q-Sort, Intuitionistic Fuzzy, and Color Interpretation of Fuzzy Degrees

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1. Fuzzy techniques are needed: a brief reminder

- In many control and decision-making situations, there are a few best specialists:
 - the best medical doctors who excel in diagnosing and curing diseases,
 - the best pilots, the best machine operators, etc.
- These specialists are very good, but there are only a few of them.
- It is not possible, e.g., to make sure that every patient is treated by one of the best doctors.
- It is therefore desirable to design automated control and decision-making systems:
 - that would incorporate the knowledge of the best specialists,
 - and either perform automatically, or provide advice to other specialists.
- Most specialists are willing to share their skills and their knowledge.

2. Fuzzy techniques are needed: a brief reminder (cont-d)

- The problem is that a significant part of this knowledge is described:
 - not in precise terms – that are easy to implement in a computer
 - but by imprecise (“fuzzy”) natural-language terms like “small”, terms that that are not easy for computer to understand.
- This problem was recognized by Lofti Zadeh.
- Zadeh proposed a technique for translating such fuzzy knowledge into precise computer-understandable terms.
- Zadeh called such translation techniques *fuzzy*.
- In his original fuzzy technique, each fuzzy term is described by assigning:
 - to each possible value x of the corresponding quantity,
 - a degree – e.g., on a scale from 0 to 1 – that describes to what extent x has this property,
 - e.g., to what extent x is small.

3. Fuzzy techniques are needed: a brief reminder (cont-d)

- Most people do not have trouble describing such degrees.
- This is how we all evaluate the quality of a service on a scale.
- This is how students evaluate our teaching.
- As a result, Zadeh's version of fuzzy technique led practitioners to many successful applications.

4. Remaining practical problem

- One of the remaining problems is that fuzzy degrees are somewhat subjective.
- Different people describe the same level of satisfaction by different values from the interval $[0, 1]$.
- This is a known phenomenon in polls.
- For example, it is known that:
 - people from New York City, in general, provide lower degrees
 - than people from the Southern states of the US.
- Because of this subjectivity, different experts provide different numbers for the same degree of satisfaction.

5. Remaining practical problem (cont-d)

- As a result, the decisions made by the fuzzy-based systems depend:
 - not only on the quality of the expert's opinion – as we would like to
 - but also on in what US state the expert lives.
- It does not make sense that:
 - we may get two different medical decisions based on the similar knowledge of two equally qualified medical doctors
 - from two different geographic areas.
- It is therefore desirable to make fuzzy degrees less subjective, so that:
 - for the fuzzy systems based on similar knowledge of two equally qualified doctors,
 - we would get the same automatic recommendations.

6. Calibration: a general idea

- Measuring instruments are not perfect.
- For example, a watch may be 2 minutes behind.
- In this case, to come up with more accurate time values, there is no need to mangle with the mechanism.
- It is sufficient to add 2 minutes to what the watch shows.
- Similarly, we can correct the reading if the watch fall behind 3 minutes per day.
- This is not so common for modern electronic watches, but it was happening in the past.

7. Calibration: a general idea (cont-d)

- In general, manufacturers of measuring instruments *calibrate* their instruments:
 - they try them on different values x_1, \dots, x_n of the corresponding quantity, and
 - if the instrument's reading x'_i is somewhat different from x_i ,
 - they add an automatic conversion of each measured value x'_i into x_i .

8. Let us apply the same idea to fuzzy techniques

- It is therefore reasonable to apply a similar procedure for fuzzy; namely:
 - for each of several values $x_1 < \dots < x_n$ from the interval $[0, 1]$
 - we can prepare several situations in which most people agree that they correspond to degree x_i .
- Thus experts may be able to better align relate their numerical degree with the community's choice.
- There is no need to select too many such degrees.
- Indeed, according to the well-known seven-plus-minus-two law, people usually classify everything into 5-to-9 categories, on average into 7.
- Thus, it makes sense to select 7 different degrees on the interval $[0, 1]$.
- It makes sense to have them equally distributed on the interval $[0, 1]$, so these degrees are:
$$0/6 = 0, 1/6, 2/6 = 1/3, 3/6 = 1/2, 4/6 = 2/3, 5/6, \text{ and } 6/6 = 1.$$

9. Let us apply the same idea to fuzzy techniques (cont-d)

- This is actually used in applications of fuzzy.
- The corresponding degrees correspond to

Strongly Disagree, Moderately Disagree, Slightly Disagree,
Neutral,
Slightly Agree, Moderately Agree, and Strongly Agree.

- It is desirable to make it easier to analyze – and to explicitly reveal the symmetry between degrees of agreeing and degrees of disagreeing.
- So, instead of the interval $[0, 1]$, usually, an interval $[-3, 3]$ is used.
- In this scale, the seven degrees are:

$-3, -2, -1, 0, 1, 2, \text{ and } 3.$

10. Q-sort: a brief description

- One area where such an expert calibration is used is pedagogy.
- In pedagogy, it is definitely important to make sure that:
 - when several teachers teach different sections of the same class,
 - their opinion of partially correct answers – and thus, their grades for partially correct answers – are similar.
- The most widely used example of such calibration is known as *Q-sort*.
- Interestingly, instead of 7 examples, it turned out to be more effective to provide an expert with 16 examples:
 - one example for each of the degrees 3 and -3 ;
 - two examples for each of the degrees 2 and -2 ;
 - three examples for each of the degrees 1 and -1 ; and
 - four examples for degree 0.

11. Q-sort: a brief description (cont-d)

- A natural question is: why is this 16-examples techniques most empirically effective?
- In this talk, we provide a possible answer to this question.

12. Analysis of this problem and the resulting explanation

- Intuitively, positive degrees 1, 2, and 3 mean that we have arguments in favor of the corresponding statement.
- Similarly, negative degrees mean that we have arguments in favor of its negation.
- Crudely speaking, we can say that:
 - the degree 0 means that we have no arguments for or against a statement,
 - the degree 1 means that we have one argument for,
 - the degree 2 means that we have two arguments for – which makes our opinion stronger, and
 - the degree 3 means that we have 3 arguments for – the strongest case.
- Similarly, the degree $-n$ means that we have n arguments against the statement.

13. Analysis of this problem and the resulting explanation (cont-d)

- Often, we have both arguments for and against a statement.
- If we have one argument for and one argument against, they kind of cancel each other, and we remain neutral.
- In general, if we have n arguments for and m arguments against and $m = n$, they cancel each other and the resulting degree is 0.
- If we have n arguments for and m argument against, and $n > m$:
 - then m pairs of for-against arguments cancel each other,
 - and the resulting degree corresponds to $n - m$ remaining positive arguments, i.e., to the degree $n - m$.

14. Analysis of this problem and the resulting explanation (cont-d)

- Similarly, if we have n arguments for and m argument against, and $n < m$:
 - then n pairs of for-against arguments cancel each other,
 - and the resulting degree corresponds to $m - n$ remaining negative arguments, i.e., to the degree $-(m - n)$.
- One can see that in all three possible cases ($n = m$, $n > m$, and $n < m$), the resulting degree is equal to $n - m$.
- From this viewpoint, each of the 7 degrees can come from several possible situations.
- For example, 0 can come from no arguments at all or from the case when we have equal number of positive and negative arguments.
- How many situations correspond to each degree?

15. Analysis of this problem and the resulting explanation (cont-d)

- Degrees 3 and -3 correspond to only one situation each:

$$3 = 3 - 0 \text{ and } -3 = 0 - 3.$$

- Degrees 2 and -2 correspond to 2 situations each:

$$2 = 2 - 0 = 3 - 1 \text{ and } -2 = 0 - 2 = 1 - 3.$$

- Degrees 1 and -1 correspond to 3 different situations each:

$$1 = 1 - 0 = 2 - 1 = 3 - 2 \text{ and } -1 = 0 - 1 = 1 - 2 = 2 - 3.$$

- Finally, the degree 0 corresponds to 4 possible situations:

$$0 = 0 - 0 = 1 - 1 = 2 - 2 = 3 - 3.$$

16. Analysis of this problem and the resulting explanation (cont-d)

- This is exactly what we observe in Q-sort – so this is probably why Q-sort is so efficient:
 - it provides not only a calibration of 7 different degrees,
 - it also provides a calibration of a more refined distinction between different possible situations leading to the same degree.

17. This explanation is related to intuitionistic fuzzy degrees

- The above description of different situations corresponding to the same degree is not new.
- It was exactly the idea that motivated the design, by Krassimir Atanassov, of intuitionistic fuzzy logics.
- Atanassov noticed that in the traditional fuzzy technique, the same neutral degree 0.5 corresponds to two different situations:
 - when we know nothing about a statement, and
 - when we have an equal number of arguments for and against this statement.

18. This explanation is related to intuitionistic fuzzy degrees (cont-d)

- So, he suggested to use, instead of a single degree, two degrees:
 - a positive degree describing the strength of the arguments for a statement and
 - a negative degree describing the strength of the arguments against.
- From this viewpoint, Q-sort provides a calibration of both positive and negative degrees.

19. Color interpretation

- We can also interpret the above explanation in terms of the color interpretation of fuzzy degrees.
- In this interpretation, 0 means white, and 1, 2, and 3 means three basic colors, e.g., sorted by the energy of their waves:

$$1 = \text{red (R)} < 2 = \text{green (G)} < 3 = \text{blue (B)}.$$

- Each negative degree $-n$ is naturally interpreted as a complementary color – with which the corresponding color n makes white:

$$-1 = BG, -2 = RB, -3 = RG.$$

- Some of these degrees can be obtained by combining different positive and negative degrees.
- This corresponds to the fact that the same visual color can be formed in different ways.

20. Color interpretation (cont-d)

- For example, white can be obtained:
 - by combining two different white colors
 - or by combining one of the 3 basic colors with its complementary color.
- This may be a way to extend:
 - color interpretation of traditional fuzzy degrees – that was successfully used in many applications,
 - to a similar interpretation of intuitionistic fuzzy degrees.

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