Increased Climate Variability Is More Visible Than Global Warming: A General System-Theory Explanation

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1. Outline

- Global warming is a statistically confirmed long-term phenomenon.
- Somewhat surprisingly, its most visible consequence is:
  - not the warming itself but
  - the increased climate variability.
- In this talk, we explain why increased climate variability is more visible than the global warming itself.
- In this explanation, use general system theory ideas.

2. Formulation of the Problem

- *Global warming* usually means statistically significant long-term increase in the average temperature.
- Researchers have analyzed the expected future consequences of global warming:
  - increase in temperature,
  - melting of glaciers,
  - raising sea level, etc.
- A natural hypothesis was that at present, we would see the same effects, but at a smaller magnitude.
- This turned out not to be the case.
- Some places do have the warmest summers and the warmest winters in record.
- However, other places have the coldest summers and the coldest winters on record.

3. Formulation of the Problem (cont-d)

- What we actually observe is unusually high deviations from the average.
- This phenomenon is called *increased climate variability*.
- A natural question is: why is increased climate variability more visible than global warming?
- A usual answer is that the increased climate variability is what computer models predict.
- However, the existing models of climate change are still very crude.
- None of these models explains why temperature increase has slowed down in the last two decades.
- It is therefore desirable to provide more reliable explanations.

4. A Simplified System-Theory Model

- Let us consider the simplest model, in which the state of the Earth is described by a single parameter x.
- In our case, x can be an average Earth temperature or the temperature at a certain location.
- We want to describe how x changes with time.
- In the first approximation, $\dfrac{dx}{dt} = u(t)$, where $u(t)$ are external forces.
- We know that, on average, these forces lead to a global warming, i.e., to the increase of $x(t)$.
- Thus, the average value u_0 of $u(t)$ is positive.
- We assume that the random deviations $r(t) \stackrel{\text{def}}{=} u(t) - u_0$ are i.i.d., with some standard deviation σ_0.

5. Towards the Second Approximation

- Most natural systems are resistant to change: otherwise, they would not have survived.
- So, when $y \stackrel{\text{def}}{=} x - x_0 \neq 0$, a force brings y back to 0: $\dfrac{dy}{dt} = f(y)$; $f(y) < 0$ for $y > 0$, $f(y) > 0$ for $y < 0$.
- Since the system is stable, y is small, so we keep only linear terms in the Taylor expansion of $f(y)$:

$$f(y) = -k \cdot y, \text{ so } \frac{dy}{dt} = -k \cdot y + u_0 + r(t).$$

- Since this equation is linear, its solution can be represented as $y(t) = y_s(t) + y_r(t)$, where

$$\frac{dy_s}{dt} = -k \cdot y_s + u_0; \quad \frac{dy_r}{dt} = -k \cdot y_r + r(t).$$

- Here, $y_s(t)$ is the *systematic* change (global warming).
- $y_r(t)$ is the *random* change (climate variability).

6. An Empirical Fact That Needs to Be Explained

- At present, the climate variability becomes more visible than the global warming itself.
- In other words, the ratio $y_r(t)/y_s(t)$ is much higher than it will be in the future.
- The change in y is determined by two factors:
  - the external force $u(t)$ and
  - the parameter k that describes how resistant is our system to this force.
- Some part of global warming may be caused by the variations in Solar radiation.
- Climatologists agree that global warming is mostly caused by greenhouse effect etc., which lowers resistance k.
- What causes numerous debates is which proportion of the global warming is caused by human activities.

7. An Empirical Fact to Be Explained (cont-d)

- Since decrease in k is the main effect, in the 1st approximation, we consider only this effect.
- In this case, we need to explain why the ratio $y_r(t)/y_s(t)$ is higher now when k is higher.
- To gauge how far the random variable $y_r(t)$ deviates from 0, we can use its standard deviation $\sigma(t)$.
- So, we fix values u_0 and σ_0, st. dev. of $r(t)$.
- For each k, we form the solutions $y_s(t)$ and $y_r(t)$ corresponding to $y_s(0) = 0$ and $y_r(0) = 0$.
- We then estimate the standard deviation $\sigma(t)$ of $y_r(t)$.
- We want to prove that, when k decreases, the ratio $\sigma(t)/y_s(t)$ also decreases.

8. Estimating the Systematic Deviation $y_s(t)$

- We need to solve the equation $\dfrac{dy_s}{dt} = -k \cdot y_s + u_0$.
- If we move all the terms containing $y_s(t)$ to the left-hand side, we get $\dfrac{dy_s(t)}{dt} + k \cdot y_s(t) = u_0$.
- For an auxiliary variable $z(t) \stackrel{\text{def}}{=} y_s(t) \cdot \exp(k \cdot t)$, we get

$$\frac{dz(t)}{dt} = \frac{dy_s(t)}{dt} \cdot \exp(k \cdot t) + y_s(t) \cdot \exp(k \cdot t) \cdot k =$$

$$\exp(k \cdot t) \cdot \left(\frac{dy_s(t)}{dt} + k \cdot y_s(t)\right).$$

- Thus, $\dfrac{dz(t)}{dt} = u_0 \cdot \exp(k \cdot t)$, so $z(t) = u_0 \cdot \dfrac{\exp(k \cdot t) - 1}{k}$, and

$$y_s(t) = \exp(-k \cdot t) \cdot z(t) = u_0 \cdot \frac{1 - \exp(-k \cdot t)}{k}.$$

9. Estimating the Random Component $y_r(t)$

- For the random component, we similarly get

$$y_r(t) = \exp(-k \cdot t) \cdot \int_0^t r(s) \cdot \exp(k \cdot s)\, ds, \text{ so}$$

$$y_r(t)^2 = \exp(-2k \cdot t) \cdot \int_0^t ds \int_0^t dv\, r(s) \cdot r(v) \cdot \exp(k \cdot s) \cdot \exp(k \cdot v),$$

$$\text{and } \sigma^2(t) = E[y_r(t)^2] =$$

$$\exp(-2k \cdot t) \cdot \int_0^t ds \int_0^t dv\, E[r(s) \cdot r(v)] \cdot \exp(k \cdot s) \cdot \exp(k \cdot v).$$

- Here, $E[r(s) \cdot r(v)] = E[r(s)] \cdot E[r(v)] = 0$ and $E[r^2(s)] = \sigma_0^2$, so

$$\sigma^2(t) = E[y_r(t)^2] = \exp(-2k \cdot t) \cdot \int_0^t ds\, \sigma_0^2 \cdot \exp(k \cdot s) \cdot \exp(k \cdot s).$$

- Thus, $\sigma^2(t) = \sigma_0^2 \cdot \dfrac{1 - \exp(-2k \cdot t)}{2k}$.

10. Analyzing the Ratio $\sigma(t)/y_s(t)$

- $\sigma^2(t) = \sigma_0^2 \cdot \dfrac{1 - \exp(-2k \cdot t)}{2k}$, $y_s(t) = u_0 \cdot \dfrac{1 - \exp(-k \cdot t)}{k}$.
- Thus, $S(t) \stackrel{\text{def}}{=} \dfrac{\sigma^2(t)}{y_s^2(t)} = \dfrac{\sigma_0^2}{u_0^2} \cdot \dfrac{(1 - \exp(-2k \cdot t)) \cdot k^2}{2k \cdot (1 - \exp(-k \cdot t))^2}$.
- Here, $1 - \exp(-2k \cdot t) = (1 - \exp(-k \cdot t)) \cdot (1 + \exp(-k \cdot t))$, so $S(t) = \dfrac{\sigma_0^2}{u_0^2} \cdot \dfrac{(1 + \exp(-k \cdot t)) \cdot k}{2 \cdot (1 - \exp(-k \cdot t))}$.
- When the k is large, $\exp(-k \cdot t) \approx 0$, and $S(t) \approx \dfrac{\sigma_0^2}{u_0^2} \cdot \dfrac{k}{2}$.
- This ratio clearly decreases when k decreases.
- So, when the Earth's resistance k will decrease, the ratio $\sigma(t)/y_s(t)$ will also decrease.
- Thus, we will start observing mainly the direct effects of global warming – unless we do something about it.

11. Discussion

- We made a simplifying assumption that the climate system is determined by a single parameter x (or y).
- A more realistic model is when the climate system is determined by several parameters $y_1, \ldots, y_n$.
- In this case, in the linear approximation, the dynamics is described by a system of linear ODEs

$$\frac{dy_i}{dt} = -\sum_{j=1}^{n} a_{ij} \cdot y_j(t) + u_i(t).$$

- In the generic case, all eigenvalues λ_k of the matrix a_{ij} are different.
- In this case, a_{ij} can be diagonalized by using the linear combinations $z_k(t)$ corresponding to eigenvectors:

$$\frac{dz_k}{dt} = -\lambda_k \cdot z_k(t) + \widetilde{u}_k(t).$$

12. Discussion (cont-d)

- Reminder: we have a system of equations

$$\frac{dz_k}{dt} = -\lambda_k \cdot z_k(t) + \widetilde{u}_k(t).$$

- For each of these equations, we can arrive at the same conclusion:
  - the current ratio of the random to systematic effects is much higher
  - than it will be in the future.
- So, our explanations holds in this more realistic model as well.

13. Acknowledgments

This work was supported in part by the US National Science Foundation grants:

- HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and
- DUE-0926721.

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