

Computation in Quantum Space-Time Can Lead to a Super-Polynomial Speedup

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1. Which Problems Are Feasible: Brief Reminder

- *In theoretical computer science*: researchers usually distinguish between
 - problems that can be solved in polynomial time, i.e., in time $\leq P(n)$ where n is input length, and
 - problems that require more computation time.
- *Terminology*:
 - problems solvable in polynomial time are usually called *feasible*,
 - while others are called *intractable*.
- *Warning*: this association is not perfect.
- *Example*: an algorithm that requires $10^{100} \cdot n$ steps is
 - polynomial time, but
 - not practically feasible.

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2. Is Speed Up Possible?

- *Problem:* some problems are intractable – i.e., require algorithms which are too slow (intractable).
- *Clarification:* they are slow when we use the physical processes which are currently used in computers.
- *Natural idea:* use new physical processes, processes that have not been used in modern computers.
- *Question:* is it possible to make computations drastically faster?
- *Reformulation:* is it possible to make intractable problems feasible?
- *This may happen:* if a physical process provides a super-polynomial (= faster than polynomial) speed-up.

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3. Quantum Computing

- *Question* (reminder): find physical processes that would make computations drastically faster.
- *Most active* research in this direction – quantum computing.
- *Fact*: quantum processes can speed up computations.
- *Example*: Grover's algorithm searches in an un-sorted list of size N in time \sqrt{N} .
- *Application*: to problems that can be solved by $N = 2^n$ time exhaustive search.
- *Example*: SAT – given a propositional formula $F(x)$, find $x = (x_1, \dots, x_n)$ s.t. $F(x)$ holds.
- *Exhaustive search*: try all 2^n possible combinations of $x_i \in \{false, true\}$.

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4. Quantum Computing: Limitations

- *Reminder*: SAT under quantum computing.
- *Grover's algorithm*: reduces the computation time from $N = 2^n$ to

$$\sqrt{N} = \sqrt{2^n} = 2^{n/2}.$$

- *Limitation*: this is still a polynomial-time speed-up:
 - let $T_c(n)$ be non-quantum time, then quantum time is $T_q(n) = \sqrt{T_c(n)}$;
 - when $T_q(n)$ is polynomial, so is $T_c(n) = T_q^2(n) :-)$
- *Fact*: some known quantum algorithms are exponentially faster than the best known non-quantum ones.
- *Example*: Shor's algorithm for factoring large integers.
- *Limitation*: it is not clear whether a similar fast non-quantum algorithm is possible.
- The only *proven* quantum speed-ups are polynomial.

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5. Potential Use of Curved Space-Time

- *Parallelization* – a natural source of speed-up.
- *Claim:* in Euclidean space-time, parallelization only leads to a polynomial speed-up.
- *Fact:* the speed of all the physical processes is bounded by the speed of light c .
- *Conclusion:* in time T , we can only reach computational units at a distance $\leq R = c \cdot T$.
- The volume $V(R)$ of this area (inside of the sphere of radius $R = c \cdot T$) is proportional to $R^3 \sim T^3$.
- So, we can use $\leq V/\Delta V \sim T^3$ computational elements.
- *Interesting:* in Lobachevsky space-time,
$$V(R) \sim \exp(R) \gg \text{Polynomial}(R).$$
- Hence, we can fit more processors – and thus get a drastic speed-up.

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6. Quantum Space-Time Models

- *So far*: we had two separate approaches:
 - use of quantum effects, and
 - use of curved space-time
- *In physics*: quantum and space-time effects are related: via quantization of space-time.
- *Natural idea*: combine the two approaches.
- *Specifics*: how quantum effects affect space-time:
 - *Heisenberg's Uncertainty Principle*: in regions of size ε , energy uncertainty is $\Delta E \sim \hbar \cdot \varepsilon^{-1}$;
 - so, when size is ε , a lot of energy enters the region;
 - this energy *curves* space-time;
 - hence, on a small scale, space-time is very curved (“foam”-like).

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7. Quantum Space-Time Models (cont-d)

- *Reminder:* all fluctuations in area of size ε have energy

$$E \sim \hbar \cdot \varepsilon^{-1}.$$

- Energy ΔE of a single fluctuation:

– according to Einstein's General Relativity, action is $L = \int R dV dt$;

– action is energy times time, hence

$$\Delta E \sim \int R dV \approx R \cdot V;$$

– for a fluctuation of size $c \cdot \varepsilon$ ($c \approx 1$), volume is $V \sim \varepsilon^3$ and curvature is $R \sim \varepsilon^{-2}$;

– hence, $\Delta E \sim \varepsilon$.

- *Conclusion:* the total number of such fluctuations is

$$E/\Delta E \sim \hbar \cdot \varepsilon^{-2}.$$

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8. Geometry of Quantum Space-Time Models

- *Reminder:* in each ε -size area, there are $n \sim \hbar \cdot \varepsilon^{-2}$ of $(c \cdot \varepsilon)$ -fluctuations.
- *Question:* how many processors $N(\varepsilon)$ of size ε can we fit in a given region?
- We can have one proc. on each of these fluctuations:

$$N(c \cdot \varepsilon) \approx \hbar \cdot \varepsilon^{-2} \cdot N(\varepsilon).$$

- $N(1) \approx 1$;
 - $N(c) \approx \hbar \cdot c^{-2}$;
 - $N(c^2) \approx \hbar \cdot c^{-4} \cdot N(c) = \hbar^2 \cdot c^{-(2+4)}$;
 - $N(c^3) \approx \hbar \cdot c^{-6} \cdot N(c^2) = \hbar^3 \cdot c^{-(2+4+6)}$;
 - ...
 - $N(c^k) \approx \hbar^k \cdot c^{-(2+4+\dots+2k)}$.
- Here, $2 + 4 + \dots + 2k = 2 \cdot (1 + 2 + \dots + k) = 2 \cdot \frac{k \cdot (k + 1)}{2} \approx k^2$, so $N(c^k) \approx \hbar^k \cdot c^{-k^2}$.

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9. Using Quantum Space-Time (ST) Models in Computations

- For the same technological level ε , we compare:
 - parallel computations in non-quantum ST, and
 - parallel computations in quantum ST.
- *Non-quantum ST*: $N_n(\varepsilon) \sim \frac{V_0}{\varepsilon^3} \sim \varepsilon^{-3}$, so $\varepsilon \sim N_n^{-1/3}$.
- *Quantum ST*: we know that $N_q(c^k) \approx \hbar^k \cdot c^{-k^2}$.
- To get $N_q(\varepsilon)$, take k s.t. $c^k = \varepsilon$, i.e., $k \sim \ln(\varepsilon)$, then

$$N_q(\varepsilon) \sim \exp(\alpha \cdot \ln^2(\varepsilon)).$$

- Substituting $\varepsilon \sim N_n^{-1/3}$ and $\ln(\varepsilon) \sim \ln(N_n)$, we get

$$N_q \sim \exp(\beta \cdot \ln^2(N_n)) = N_n^{\beta \cdot \ln(N_n)}.$$

- *Fact*: this expression grows faster than any polynomial.

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