# Why Ratio Bias?

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## 1. Formulation of the problem

- In general, people prefer alternative with higher probability of success – and lower probability of failure.
- However, there are exceptions to this reasonable idea.
- Namely, when the same probability comes from a larger sample, people prefer this alternative.
- For example, suppose that we have two alternatives:
  - one of them was successful 90 times out of 100, while
  - the second one was successful 9 times out of 10.
- The probability of success is the same in both cases, but most people prefer the second alternative.
- This is especially true in health-related decisions.
- This phenomenon is known as the *ratio bias*.

## 2. Formulation of the problem (cont-d)

- It is an important case of more general *numerator bias*, when people pay more attention to the numerator than to the ratio.
- How can we explain this bias?

## 3. It is not probabilities that are equal

- We said that the probability is the same in both above cases.
- However, this is not exactly correct.
- What is the same is the frequency, and probability is, in general, somewhat different from the frequency.
- For example, a fair coin lands heads with probability 0.5.
- This means that, on average, it falls heads in half of the cases.
- However, it does not mean that if we flip the coin 100 times, it will fall heads exactly 50 times.
- This is even more clear if we flip a coin 101 times: here, exactly half would mean 50.5 times – and 50.5 is not even an integer.
- If we have an event with probability $p$, then its average frequency is indeed $p$.
- However, the frequency may deviate from probability.

## 4. How to describe the deviation of frequency from probability?

- In statistics, we usually gauge such deviation by its root mean square value $\sigma$ known as *standard deviation*:
$$\sigma = \sqrt{E[(f-p)^2]}.$$
- It is known that for repeating events with probability $p$:
$$\sigma(p) = \sqrt{\frac{p\cdot(1-p)}{n}}.$$
- In principle, deviation can be as large as possible, but the probability of deviations much larger than a few signa's is very small.
- However, in statistics, when we make decisions, we usually ignore such rare deviations.
- Thus, we assume that deviations do not exceed $k\cdot\sigma$ for some small $k$, usually $k = 2, 3, 6$.

## 5. How to describe the deviation of frequency from probability (cont-d)

- For $k = 2$, the probability of exceeding this bound is 5%, for $k = 3$, it is 0.1%, and for $k = 6$, it is $10^{-6}$%.
- So, based on the observations, we do not get the exact value of the probability.
- What we conclude is that the actual (unknown) probability is somewhere in the interval

$$[p - k \cdot \sigma, p + k \cdot \sigma] = \left[p - k \cdot \sqrt{\frac{p \cdot (1-p)}{n}}, p + k \cdot \sqrt{\frac{p \cdot (1-p)}{n}}\right].$$

- Since we do not know the exact probability, we do not know the exact value of the expected gain $p \cdot g_0$.
- All we know is that this gain is somewhere in the interval

$$[\underline{u}, \overline{u}] = [(p - k \cdot \sigma) \cdot g_0, (p + k \cdot \sigma) \cdot g_0].$$

## 6. How to make a decision under such interval uncertainty?

- When we know the exact gain of different alternatives, we select the alternative with the largest expected gain.
- How can we make a decision if we only the interval of possible gains?
- In such cases, decision theory recommends to use so-called Hurwicz criterion.
- Namely, we pick some value $\alpha$ between 0 and 1, and select the alternative for which the following combination is the largest:

$$u = \alpha \cdot \overline{u} + (1 - \alpha) \cdot \underline{u}.$$

- The coefficient $\alpha$ is known as optimism-pessimism coefficient.

## 7. Both pure optimism and pure pessimism are not good

- When $\alpha = 1$, we have pure optimism: $u = \overline{u}$.
- So, the decision maker only takes into account the best possible case and ignores the rest.
- For example, if he/she drives to NMSU, he/she thinks that there will no roadwork, no accidents, no traffic jams.
- As a result, this person is always late.
- When $\alpha = 0$, we have pure pessimism: $u = \underline{u}$.
- Such a person never takes a flight, because in the worst case, a plane may crash, etc.
- A realistic person has $\alpha$ between 0 and 1.

## 8. In medical cases, the emphasis is usually on not harming the patient

- In medical cases, decision makers tend to be cautious.
- Unless it is a critical situation, a medicine is not prescribed unless the doctor is absolutely sure that it will not harm the patient.
- In Hurwicz terms, it means that in medical situations, we pay more attention to the worst case than to the best case.
- In other words, we have $1-\alpha>\alpha$, i.e., $1-2\alpha>0$.
- For this $\alpha$, the Hurwicz criterion takes the form

$$u=\alpha\cdot(p+k\cdot\sigma+(1-\alpha)\cdot(p-k\cdot\sigma)=p-(1-2\alpha)\cdot k\cdot\sigma=$$

$$p-(1-2\alpha)\cdot k\cdot\sqrt{\frac{p\cdot(1-p)}{n}}.$$

## 9. This explains the ratio bias

- One can see that for a fixed $p$, when $n$ increases, $\sigma$ decreases, and thus, $u$ increases.
- Between the two alternatives with the same probability $p$, we select the one for which $n$ is larger.
- This is exactly what people do – so we have explained the ratio bias.

## 10. Acknowledgments

This work was supported by:

- the AT&T Fellowship in Information Technology,
- the Institute for Risk and Reliability, Leibniz Universitaet Hannover, Germany,
- the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Focus Program SPP 100+ 2388, Grant Nr. 501624329,
- the European Union under the project ROBOPROX (No. CZ.02.01.01/00/22 008/0004590),
- the Center of Excellence in Econometrics, Faculty of Economics, Chiang Mai University, Thailand,
- the Ho Chi Minh City University of Banking, Vietnam, and
- Thang Long University, Hanoi, Vietnam.