# Spiral Arms Around a Star: Geometric Explanation

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## 1. Formulation of the problem

- Spiral arms are typical in galaxies.
- This is what our own Galaxy consists of.
- There are many physical theories that explain the appearance of logarithmic spiral arms in galaxies.
- Recently, very similar spiral arms were discovered around a star.
- Problem:
  - the geometric shape is similar,
  - but, because of the different scale, galaxy-related explanations cannot be directly applied.
- So, a natural question is: how to explain the appearance of spiral arms around a star?

## 2. How are spiral arms in galaxies explained?

- For galaxies, several dozen different physical theories have been proposed that explain the same spiral shape.
- All these theories, based on completely different physics, successfully explain the same shape.
- This led researchers to conclude that this shape must have a simple geometric explanation.
- Such an explanation has indeed been proposed.
- Let us recall this explanation.

## 3. Initial symmetries and inevitability of symmetry violations

- The distribution of matter close to the Big Bang was practically uniform and homogeneous.
- Thus, this distribution was invariant with respect to shifts, rotations, and scalings.
- However, such a distribution is unstable.
- If at some point, density increases:
  - matter will be attracted to this point, and
  - the disturbance will increase.
- Thus, the original symmetry will be violated.

## 4. Which symmetry violations are more probable?

- According to statistical physics, it is more probable to go from a symmetric state to a state where some symmetries are preserved.
- For example, typically, when heating, the matter does go directly:
  - from a highly symmetric crystal state
  - to the completely asymmetric gas state.
- It first goes through intermediate liquid state, where some symmetries are preserved.
- The more symmetries are preserved, the more probable the transition.
- From this viewpoint, it is most probable that matters forms a state with the largest group of symmetries.

## 5. Resulting shapes consists of orbits of symmetry groups

- Here:
  - if there is a disturbance at some point $a$, and
  - the situation is invariant with respect to some transformation $g$,
  - this means that there is a disturbance at the point $g(a)$ as well.
- So, with each point $a$, the resulting shape contains all the points

$$G(a) = \{g(a) : g \in G\}.$$

- The set $G(a)$ is known as an *orbit* of the group $G$.
- For example, if $G$ is the group of all rotations around a point, then $G(a)$ is the sphere containing $a$.

## 6. Original symmetry group

- In the beginning, we have the following 1-dimensional families of basic transformations:
  - three families of shifts $x \mapsto x + a$ – in all three dimensions,
  - three families of rotations $x \mapsto Rx$ – around all three axes, and
  - one family of scalings $x \mapsto \lambda \cdot x$.
- We can combine them, so we get a 7-dimensional symmetry group.

## 7. What is the next shape?

- What is the shape with the largest symmetry?
- If we have all three shifts, then from each point, we can get to every other point.
- In this case, the whole 3-D space is the shape.
- Thus, for perturbation shapes, we can have at most two families of shifts.
- If we apply two families of shifts to a point, we get a plane.
- The plane also has:
  - one family of rotations inside the plane, and
  - one family of scalings.
- Thus, the plane has a 4-dimensional symmetry group.

## 8. What is the next shape (cont-d)

- If we have one family of shifts, we get a straight line.
- It also has rotations around it and scaling.
- So, a straight line has a 3-dimensional symmetry group.
- If we have no shifts, but all rotations, then we get a sphere.
- A sphere has a 3-dimensional symmetry group.
- So far, the most symmetric shape is the plane.
- A detailed analysis of all possible symmetry groups confirms this.
- So, the most probable first perturbation shape is a plane.
- This is in accordance with astrophysics, where such a proto-galaxy shape is called a pancake.

## 9. What next after a pancake?

- The planar shape is still not stable.
- So, the symmetry group decreases further.
- From the 2-D shape of a plane, we go to a symmetric 1-D shape.
- The generic form of a 1-D group is exactly logarithmic spiral.
- In polar coordinates, it has the form $r = a \cdot \exp(k \cdot \theta)$.
- For every two its points $(r, \theta)$ and $(r', \theta')$, there is spiral's symmetry transforming $(r, \theta)$ and $(r', \theta')$:
  - first, we rotate by $\delta = \theta' - \theta$,
  - then, we scale $r \mapsto \exp(k \cdot \delta) \cdot r$.
- This is what we observe in galaxies.

## 10. What is after spirals?

- What will happen next?
- The spiral is also unstable.
- So, we go from the continuous 1-D symmetry group to a discrete one.
- In the resulting shape, we have points whose distances from a central point form a geometric progression $r_n = r_0 \cdot q^n$.
- Interestingly, this is exactly the Titius-Bode formula describing planets' distances from the Sun.
- The only exception to this formula is that after Earth and Mars, this formula gets a distance to the asteroid belt between Mars and Jupiter.
- Astrophysicists believe that there was a proto-planet there torn apart by Jupiter's gravity, and asteroids are its remainder.

## 11. Resulting explanation of spiral arms around a star

- A general geometric analysis shows that at some point, we should reach a spiral shape.
- This explains the observed spiral arms around a star.
- This also explains why such arms are rare and were never observed before.
- In our Solar system – and in other places – we have moved to the next stage, of planets following Titius-Bode law.

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