An Arbitrary Preference Relation Can Be Represented in Qualitative Choice Logic: A Remark

Olga Kosheleva[1], Vladik Kreinovich[2], and Aldo A. Venzor Nava[2]
Departments of [1]Teacher Education and [2]Computer Science
University of Texas at El Paso
olgak@utep.edu, vladik@utep.edu, aavenzornav@miners.utep.edu

1. Formulation of the problem

- How do we describe what we want?
- Out of several properties $P_1, \ldots, P_n$ of the desired alternatives, we may want to have some properties and no others.
- For example, when we are looking for a house, we may want:
  - either a big house (P_1) or a smaller house ($\neg P_1$) in a good school district (P_2),
  - but not in a bad neighborhood ($\neg P_3$).
- In such situations, what we want can be described by a propositional formula.
- For example, in the above case, this formula takes the form

$$(P_1 \vee (\neg P_1 \,\&\, P_2)) \,\&\, \neg P_3.$$

2. Formulation of the problem (cont-d)

- But what if such an ideal object is not available?
- To make a decision in such cases, we need to also describe preferences.
- For example, we can say

  "we want A but if A is not possible, then we should have B."

- This condition is described by $A \vec{\times} B$.
- For example, if we want to nominate a student for the best student award:
  - we may want to have a straight-A student
  - but if there are no such students, at least a student with high GPA.
- The idea of adding the new connective $\vec{\times}$ to propositional logic first appeared in Brewka et al. 2004.
- The resulting logic is called *Qualitative Choice Logic.*

3. Formulation of the problem (cont-d)

- Several other additional connectives have been proposed to describe preferences.
- These additional connectives help to speed up computations.
- A natural question is:
  - are these other connectives needed to represent human preferences
  - or, in principle, $\vec{\times}$ is sufficient?

4. Our answer

- In this talk, we provide a positive answer to this question.
- Yes, every preference relation can be represented in Qualitative Choice Logic.
- Indeed, with n properties, we can, in principle, have 2^n possible situations described by formulas $P(\varepsilon)$ of the type $P_1^{\varepsilon_1} \,\&\, \ldots \,\&\, P_n^{\varepsilon_n}$.
- Here, $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$, $\varepsilon_i \in \{-,+\}$, P_i^+ means P_i, and P_i^- means $\neg P_i$.
- A general preference relation is a strict partial order $<$ between such formulas.
- Here $a < b$ means that if both a and b are available, we prefer b.
- To describe this relation in Qualitative Choice Logic, we take all the formulas $b \vec{\times} a$ corresponding to pairs (a, b) for which $a < b$.
- One can see that this indeed leads to the desired representation.

5. References

- M. Bernreiter, J. Maly, and S. Woltran, “Choice logics and their computational properties”, *Artificial Intelligence*, 2022, Vol. 311, Paper 103755.
- G. Brewka, S. Benferhat, and D. Le Berre, “Qualitative Choice Logic”, *Artificial Intelligence*, 2004, Vol. 157, pp. 203–237.

6. Acknowledgments

- This work was supported in part by the National Science Foundation grants:
  - 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and
  - HRD-1834620 and HRD-2034030 (CAHSI Includes).
- It was also supported by the AT&T Fellowship in Information Technology.
- It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.