An Arbitrary Preference Relation Can Be Represented in Qualitative Choice Logic: A Remark

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1. Formulation of the problem

- How do we describe what we want?
- Out of several properties $P_1, \ldots, P_n$ of the desired alternatives, we may want to have some properties and no others.
- For example, when we are looking for a house, we may want:
  - either a big house ($P_1$) or a smaller house ($\neg P_1$) in a good school district ($P_2$),
  - but not in a bad neighborhood ($\neg P_3$).
- In such situations, what we want can be described by a propositional formula.
- For example, in the above case, this formula takes the form
  \[(P_1 \lor (\neg P_1 \land P_2)) \land \neg P_3.\]
2. Formulation of the problem (cont-d)

- But what if such an ideal object is not available?
- To make a decision in such cases, we need to also describe preferences.
- For example, we can say

  “we want A but if A is not possible, then we should have B.”

- This condition is described by $A \vec{\times} B$.
- For example, if we want to nominate a student for the best student award:
  - we may want to have a straight-A student
  - but if there are no such students, at least a student with high GPA.
- The idea of adding the new connective $\vec{\times}$ to propositional logic first appeared in Brewka et al. 2004.
- The resulting logic is called *Qualitative Choice Logic*. 
3. Formulation of the problem (cont-d)

- Several other additional connectives have been proposed to describe preferences.
- These additional connectives help to speed up computations.
- A natural question is:
  - are these other connectives needed to represent human preferences
  - or, in principle, $\times$ is sufficient?
4. Our answer

- In this talk, we provide a positive answer to this question.
- Yes, every preference relation can be represented in Qualitative Choice Logic.
- Indeed, with $n$ properties, we can, in principle, have $2^n$ possible situations described by formulas $P(\varepsilon)$ of the type $P_{1}^{\varepsilon_1} \land \ldots \land P_{n}^{\varepsilon_n}$.
- Here, $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$, $\varepsilon_i \in \{-, +\}$, $P_i^+$ means $P_i$, and $P_i^-$ means $\neg P_i$.
- A general preference relation is a strict partial order $<$ between such formulas.
- Here $a < b$ means that if both $a$ and $b$ are available, we prefer $b$.
- To describe this relation in Qualitative Choice Logic, we take all the formulas $b \prec a$ corresponding to pairs $(a, b)$ for which $a < b$.
- One can see that this indeed leads to the desired representation.
5. References


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