Why Earthquake Statistics Vary with Fault Size: An Invariance-Based Qualitative Explanations

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1. Formulation of the problem

- Earthquakes can be devastating, so researchers have been studying them starting with ancient times.
- The main emphasis have always been on areas where the strongest, the most devastating earthquakes are possible.
- According to modern geosciences, earthquakes are mainly occurring in the close vicinity of faults.
- Faults are places where there is discontinuity – between tectonic plates, between terranes, etc.
- In general, the larger the fault, the more potential energy it contains, so the stronger the earthquakes.
- Usually, a strong earthquake is followed by a sequence of weaker earthquakes.
- Their strength $s(t)$ decreases with time t as a power law $s(t) \approx C \cdot t^{-a}$ for some C and a.

2. Formulation of the problem (cont-d)

- With more accurate measuring instruments, it is now possible to study smaller-size earthquakes as well.
- These earthquakes correspond to smaller-size faults.
- Researchers expected that the resulting sequences of aftershocks would follow a similar power law.
- However, surprisingly, it turned out that for such faults, the strength of the follow-up earthquakes does not decrease with time at all.
- Recent research provides an explanation based on detailed geophysical model.
- In this talk, we show that – at least on the qualitative level – this phenomenon can be explained based on general invariance ideas.

3. Our explanation

- Numerical values of physical quantities depend on the choice of the measuring units and on the choice of the starting point.
- In many cases, there is no preferred measuring unit.
- So it makes sense to conclude that:
  - the dependence $y = f(x)$ between the quantities
  - should not depend on what unit we select for measuring these quantities.
- When we change a measuring unit to a new one which is λ times smaller, all the numerical values are multiplied by λ: $x \to x' = \lambda \cdot x$.
- For example, 2 m becomes $2 \cdot 100 = 200$ cm.

4. Our explanation (cont-d)

- Of course, when we change a measuring unit for x, we need to appropriately change a measuring unit for y.
- For example, the formula $y = x^3$ for the volume of a cube does not depend on the units.
- However:
  - when we change the measuring unit for the cube's linear size x,
  - we need to appropriately change the unit for measuring the volume y: from m^3 to cm^3.
- In general, for every $\lambda > 0$, there exists a $\mu > 0$ (depending on λ) for which:
  - once we have $y = f(x)$, we should also have $y' = f(x')$,
  - where $x' = \lambda \cdot x$ and $y' = \mu(\lambda) \cdot y$.
- It is known that the only measurable functions $f(x)$ satisfying this property are power laws.

5. Our explanation (cont-d)

- This explains the larger-faults power law.
- What about starting points?
- For strength and for many other physical quantities, there is a natural starting point – e.g., 0 for earthquake strength.
- For some other quantities like time, there is no natural starting point.
- For aftershocks following a strong earthquake, there is a natural starting point for time: the time of this strong earthquake.
- For larger-size faults, a typical strong earthquake drastically changes the geological structure, releases the stress.
- So there is a clear difference between the situations before and after the major earthquake.
- Thus, the time of this earthquake thus serves as a natural starting point for measuring time.

6. Our explanation (cont-d)

- In contrast, small earthquakes (typical for smaller faults) do not have enough power to make drastic changes.
- There is almost no difference in the geological structure before and after the earthquake.
- Thus, there is no natural starting point for time.
- If we change a starting point to a new ones t_0 moments in the past, then each numerical value t is replaced by the new value $t' = t + t_0$.
- In this case, the additional requirement that the physics should not depend on the selection of the starting point means that:
  - for every t_0, we should have an appropriate re-scaling

$$s \to s' = \mu(t_0) \cdot s$$

  - for which $s = s(t)$ would imply $s' = s(t')$.

7. Our explanation (cont-d)

- Since we know that $s(t) = C \cdot t^a$, this condition implies that $a = 0$, i.e., that $s(t)$ is a constant.
- This is exactly what we observe.

8. Bibliography

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9. Acknowledgments

- This work was supported in part by the National Science Foundation grants:
  - 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and
  - HRD-1834620 and HRD-2034030 (CAHSI Includes).
- It was also supported by the AT&T Fellowship in Information Technology.
- It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.