Why Earthquake Statistics Vary with Fault Size:
An Invariance-Based Qualitative Explanations

Jannatun N. Jotey and Vladik Kreinovich
Department of Computer Science
University of Texas at El Paso
jnjjotev@miners.utep.edu, vladik@utep.edu
1. Formulation of the problem

- Earthquakes can be devastating, so researchers have been studying them starting with ancient times.
- The main emphasis have always been on areas where the strongest, the most devastating earthquakes are possible.
- According to modern geosciences, earthquakes are mainly occurring in the close vicinity of faults.
- Faults are places where there is discontinuity – between tectonic plates, between terranes, etc.
- In general, the larger the fault, the more potential energy it contains, so the stronger the earthquakes.
- Usually, a strong earthquake is followed by a sequence of weaker earthquakes.
- Their strength $s(t)$ decreases with time $t$ as a power law $s(t) \approx C \cdot t^{-a}$ for some $C$ and $a$. 
2. Formulation of the problem (cont-d)

- With more accurate measuring instruments, it is now possible to study smaller-size earthquakes as well.
- These earthquakes correspond to smaller-size faults.
- Researchers expected that the resulting sequences of aftershocks would follow a similar power law.
- However, surprisingly, it turned out that for such faults, the strength of the follow-up earthquakes does not decrease with time at all.
- Recent research provides an explanation based on detailed geophysical model.
- In this talk, we show that – at least on the qualitative level – this phenomenon can be explained based on general invariance ideas.
3. Our explanation

- Numerical values of physical quantities depend on the choice of the measuring units and on the choice of the starting point.
- In many cases, there is no preferred measuring unit.
- So it makes sense to conclude that:
  - the dependence $y = f(x)$ between the quantities
  - should not depend on what unit we select for measuring these quantities.
- When we change a measuring unit to a new one which is $\lambda$ times smaller, all the numerical values are multiplied by $\lambda$: $x \rightarrow x' = \lambda \cdot x$.
- For example, $2 \text{ m}$ becomes $2 \cdot 100 = 200 \text{ cm}$.
4. Our explanation (cont-d)

- Of course, when we change a measuring unit for \( x \), we need to appropriately change a measuring unit for \( y \).
- For example, the formula \( y = x^3 \) for the volume of a cube does not depend on the units.
- However:
  - when we change the measuring unit for the cube’s linear size \( x \),
  - we need to appropriately change the unit for measuring the volume \( y \): from m\(^3\) to cm\(^3\).
- In general, for every \( \lambda > 0 \), there exists a \( \mu > 0 \) (depending on \( \lambda \)) for which:
  - once we have \( y = f(x) \), we should also have \( y' = f(x') \),
  - where \( x' = \lambda \cdot x \) and \( y' = \mu(\lambda) \cdot y \).
- It is known that the only measurable functions \( f(x) \) satisfying this property are power laws.
5. Our explanation (cont-d)

- This explains the larger-faults power law.
- What about starting points?
- For strength and for many other physical quantities, there is a natural starting point – e.g., 0 for earthquake strength.
- For some other quantities like time, there is no natural starting point.
- For aftershocks following a strong earthquake, there is a natural starting point for time: the time of this strong earthquake.
- For larger-size faults, a typical strong earthquake drastically changes the geological structure, releases the stress.
- So there is a clear difference between the situations before and after the major earthquake.
- Thus, the time of this earthquake thus serves as a natural starting point for measuring time.
6. Our explanation (cont-d)

- In contrast, small earthquakes (typical for smaller faults) do not have enough power to make drastic changes.

- There is almost no difference in the geological structure before and after the earthquake.

- Thus, there is no natural starting point for time.

- If we change a starting point to a new ones $t_0$ moments in the past, then each numerical value $t$ is replaced by the new value $t' = t + t_0$.

- In this case, the additional requirement that the physics should not depend on the selection of the starting point means that:
  
  - for every $t_0$, we should have an appropriate re-scaling
    
    $$ s \rightarrow s' = \mu(t_0) \cdot s $$
    
  - for which $s = s(t)$ would imply $s' = s(t')$. 
7. Our explanation (cont-d)

- Since we know that \( s(t) = C \cdot t^a \), this condition implies that \( a = 0 \), i.e., that \( s(t) \) is a constant.
- This is exactly what we observe.
8. Bibliography


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