Why Mania Leads to Risky Behavior

Mohammad T. Nazim, Monjur Bin Shams, and Vladik Kreinovich
Department of Computer Science
University of Texas at El Paso, El Paso, Texas 79968, USA
mtnazim@miners.utep.edu, mshams@miners.utep.edu, vladik@utep.edu

1. Formulation of the problem

- It is known that manic patients – the ones who view the world too positively – tend to make risky decision.
- They bet of low probability events, and overall lose.
- The same behavior – to a lesser degree – has been observed for people who are in an unusually optimistic mood.
- The more optimistic the person, the more risky this person's behavior can be.
- On the other hand, more pessimistic people is more risk-averse.
- How can we explain this empirical relation between optimism and risk-taking?

2. Our explanation

- According to decision theory:
  - when faced with the need to make a decision,
  - people (should) select an alternative for which the expected utility is the largest.
- For example, in the betting situations:
  - when we have several alternatives i with utility u_i and known probability p_i,
  - we select an alternative for which the expected utility $p_i \cdot u_i$ is the largest.
- The difficulty is that usually, we do not know the exact probabilities.
- All we know, based on the previous n observations, is the frequency f_i of the i-th outcome.

3. Our explanation (cont-d)

- The frequency is, in general, somewhat different from the probability.
- The corresponding standard deviation is equal to

$$\sigma_i = \sqrt{\frac{f_i \cdot (1 - f_i)}{n}}.$$

- Thus, with confidence 95%:
  - the only thing we can conclude about the actual (unknown) value of the probability
  - is that this value is located on the interval

$$[\underline{p}_i, \overline{p}_i] = [f_i - 2\sigma_i, f_i + 2\sigma_i].$$

- Under such interval uncertainty, decision theory recommends:
  - to select the value $\widetilde{p}_i = \alpha \cdot \overline{p}_i + (1 - \alpha) \cdot \underline{p}_i$,
  - for some value α that describes the decision maker's optimism-pessimism level.

4. Our explanation (cont-d)

- When α is close to 1, the person only takes into account the most optimistic scenario.
- When α is small, only the most pessimistic one.
- The value α is the numerical description of degree of mania or depression (in the extreme case) and of the degree of optimism in general.
- In gambling, each person selects an alternative for which $\widetilde{p}_i \cdot u_i$ is the largest, where $\widetilde{p}_i = f_i + 2(2\alpha - 1) \cdot \sigma_i$.
- We show that for an optimistic person P (with $\alpha > 0.5$):
  - if we have two alternatives i and j with $f_i < f_j$ that are equivalent to a "normal" person (for whom $\alpha = 0.5$), i.e., for which

$$f_i \cdot u_i = f_j \cdot u_j = c,$$

  - then for P, we will have $\widetilde{p}_i \cdot u_i > \widetilde{p}_j \cdot u_j$ (see Proof).

5. Proof

- Indeed, $\widetilde{p}_i \cdot u_i > \widetilde{p}_j \cdot u_j \Leftrightarrow$

$$\left(f_i + 2(2\alpha - 1)\sqrt{\frac{f_i \cdot (1 - f_i)}{n}}\right) \cdot u_i > \left(f_j + 2(2\alpha - 1)\sqrt{\frac{f_j \cdot (1 - f_j)}{n}}\right) \cdot u_j \Leftrightarrow$$

$$f_i \cdot u_i + 2(2\alpha - 1) \cdot \sqrt{\frac{f_i \cdot (1 - f_i)}{n}} \cdot u_i > f_j \cdot u_j + 2(2\alpha - 1) \cdot \sqrt{\frac{f_j \cdot (1 - f_j)}{n}} \cdot u_j.$$

- Subtracting $f_i \cdot u_i = f_j \cdot u_j$ from both sides, we get an equivalent inequality

$$2(2\alpha - 1) \cdot \sqrt{\frac{f_i \cdot (1 - f_i)}{n}} \cdot u_i \cdot u_i > 2(2\alpha - 1) \cdot \sqrt{\frac{f_j \cdot (1 - f_j)}{n}} \cdot u_j.$$

- Dividing both sides by $2(2\alpha - 1)$ and multiplying both sides by $\sqrt{n}$, we get the following equivalent inequality:

$$\sqrt{f_i \cdot (1 - f_i)} \cdot u_i > \sqrt{f_j \cdot (1 - f_j)} \cdot u_j.$$

6. Proof (cont-d)

- Here, $u_i = c/f_i$ and $u_j = c/f_j$; substituting these values, we get the following equivalent form:

$$c \cdot \sqrt{\frac{1-f_i}{f_i}} > c \cdot \sqrt{\frac{1-f_j}{f_j}}.$$

- Dividing both sides by c and squaring both sides, we have the following equivalent inequality

$$\frac{1-f_i}{f_i} > \frac{1-f_j}{f_j}, \text{ i.e., } \frac{1}{f_i} - 1 > \frac{1}{f_j} - 1.$$

- Adding 1 to both sides, we get an equivalent inequality $\dfrac{1}{f_i} > \dfrac{1}{f_j}$, which is true since $f_i < f_j$.

- Thus, the equivalent inequality $\widetilde{p}_i \cdot u_i > \widetilde{p}_j \cdot u_j$ is also true.

7. Our explanation (cont-d)

- Thus:
  - if we slightly decrease f_i – so that we get $f_i \cdot u_i < f_j \cdot u_j$, making betting on i unnecessarily risky,
  - we will still have $\widetilde{p}_i \cdot u_i > \widetilde{p}_j \cdot u_j$.
- So the optimistic person will still bet on this risky low-probability option.
- It is also possible to show that the larger α, the smaller the threshold for f_i at which the person with this α will bet on this alternative.
- For a person Q with $\alpha < 0.5$, similar arguments lead to the opposite effect:
  - even when gambling on a low probability option i (with $f_i < f_j$) makes sense for a "normal" person (for whom $f_i \cdot u_i = f_j \cdot u_j$),
  - for Q, we will have $\widetilde{p}_i \cdot u_i < \widetilde{p}_j \cdot u_j$.
- This explains why more pessimistic people are more risk-averse.

8. References

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