How to Explain Empirical Metric on the Set of Colors

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1. Formulation of the problem

- It is known that human color perception corresponds to the 3D space.
- Namely, every color that we see can be perfectly emulated by a combination of three colors.
- Researchers are also interested in how we perceive the difference between different colors.
- For this purpose, they use volunteers to estimate the distance between different colors by a number.
- There are formulas that allow us to predict, for every two close colors, the user's estimate of the distance between these two colors.
- It is desirable:
  - based on these formulas,
  - to be able to predict the subjective distance between any two colors – which are not necessarily close to each other.

2. Formulation of the problem (cont-d)

- If this was the geometric distance – e.g., distance between two locations on Earth – this would be straightforward to do.
- For each path between the two colors, we can find the total length of this path.
- We can do it by adding the lengths of all it short segments that form this path.
- Then, we define the distance $d(a, b)$ between the points a and b as the shortest length of the path that connects a and b.
- We can perform the same procedure for two colors a and b and get the length $d(a, b)$ of the shortest path that connects a and b.
- However, in contrast to the geometric distance, the resulting value $d(a, b)$ is different from the estimate $e(a, b)$ provided by humans.
- Namely, $e(a, b) \approx C \cdot \ln(d(a, b))$.
- How can we explain this empirical formula?

3. Our explanation

- On a ruler, the difference between two values is proportional to the number of different readings separating these values.
- For example, in a metric ruler, where we have readings at a millimeter distance, there are 20 readings between 1 cm and 3 cm.
- We can similarly describe our perception.
- For each value x_0 of a quantity, values x that are very close to x_0 cannot be distinguished from x_0.
- As we increase x, we will come up with the smallest value $x_1 > x_0$ that is distinguishable from x_0.
- Then, we will similarly have the smallest value $x_2 > x_1$ that is distinguishable from x_1, etc.

4. Our explanation (cont-d)

- If we start with some fixed value x_0, then:
  - a natural way for us to gauge a value $x > x_0$
  - is by the number n of such distinguishable values x_i between x_0 and x.
- We want to find the perceived distance n as a function of the actual distance x.
- For this purpose, let us analyze what will be, for each value x, the smallest value $y = f(x) > x$ which is distinguishable from x.
- There is no preferred measuring unit for distance.

5. Our explanation (cont-d)

- So it makes sense to require that:
  - the relation $y = f(x)$ remain the same if we change the unit to a new one which is λ times smaller,
  - i.e., if we replace x and y with $x' = \lambda \cdot x$ and $y' = \lambda \cdot y$.
- So, $y = f(x)$ implies that $f(\lambda \cdot x) = \lambda \cdot f(x)$.
- In particular, for $x = 1$, we get $f(\lambda) = c \cdot \lambda$, where $c = f(1)$.
- Thus, $x_1 = c \cdot x_0$, $x_2 = c \cdot x_1 = c^2 \cdot x_0$, and, in general, $x_n = c^n \cdot x_0$.
- So $x_n = x$ implies that $n = \log_c(x/x_0)$.
- This explains why the perceived distance n is proportional to the logarithm of the actual distance.

6. Reference

- R. Bujack, E. Teti, J. Miller, E. Caffrey, and T. L. Turton, "The non-Riemannian nature of perceptual color space", *Proceedings of the National Academy of Science of the USA*, 2022, Vol. 119, No. 18, Paper e2119753119, https://doi.org/10.1073/pnas.2119753119.

7. Acknowledgments

- This work was supported in part by the National Science Foundation grants:
  - 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and
  - HRD-1834620 and HRD-2034030 (CAHSI Includes).
- It was also supported by the AT&T Fellowship in Information Technology.
- It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.