Towards Decision Making Under Interval Uncertainty

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1. Formulation of the problem

- In many real-life situations, we need to make a decision.
- The quality of the decision usually depends on the value of some quantity x.
- For example, in construction:
  - the speed with which the cement hardens depends on the humidity, and
  - thus, the proportions of the best cement mix depend on the humidity.
- In practice, we often do not know the exact value of the corresponding quantity.

2. Formulation of the problem (cont-d)

- For example, in the case of the pavement:
  - while we can accurate measure the current humidity,
  - what is really important is the humidity in the next few hours.
- For this future value, at best, we only know the bounds, i.e., we only know the interval $[\underline{x}, \overline{x}]$ that contains the actual (unknown) value x.
- To select a decision, we need to select some value x_0 from this interval.
- Which value should we select?

3. Our solution

- In such situations of interval uncertainty, the ideal case is when the selected value x_0 is exactly equal to the actual value x.
- When these two values differ, i.e., when $x < x_0$ or $x > x_0$, the situation becomes worse.
- In both cases when $x < x_0$ and when $x > x_0$, we have losses, but we often have two different reasons for a loss.
- If the humidity will be larger than expected, the hardening of the cement will take longer and we will lose time (and thus, money).
- In contrast, if the humidity is lower than expected, the cement will harden too fast, and the pavement will not be as stiff as it could be.
- So we will not get a premium for a good quality road (and we may even be required to repave some road segments).
- In both cases, the larger the difference $|x - x_0|$, the larger the loss.

4. Our solution (cont-d)

- The interval $[\underline{x}, \overline{x}]$ is usually reasonable narrow, so the difference is small.
- In this case, the dependence of the loss on the difference can be well approximated by a linear expression; so:
  - when $x < x_0$, the loss is $\alpha_- \cdot (x_0 - x)$ for some α_-, and
  - when $x > x_0$, the loss is $\alpha_+ \cdot (x - x_0)$ for some α_+.
- In the first case, the worst-case loss is when x is the smallest:
$$\alpha_- \cdot (x_0 - \underline{x}).$$
- In the second case, the worst-case loss is when x is the largest:
$$\alpha_+ \cdot (\overline{x} - x_0).$$
- In general, the worst-case loss is the largest of these two:
$$w(x_0) = \max(\alpha_- \cdot (x_0 - \underline{x}), \alpha_+ \cdot (\overline{x} - x_0)).$$

5. Our solution (cont-d)

- The best-case loss is 0 – when we guessed the value x correctly.
- In this case, all we know is that the loss is somewhere between 0 and $w(x_0)$, i.e., the gain is somewhere between $\underline{g} = -w(x_0)$ and $\overline{g} = 0$.
- In such situations, decision theory recommends to use Hurwicz optimism-pessimism criterion, i.e.:
  - to select some value $\alpha > 0$ and
  - then to select an alternative for which the value $g \stackrel{\text{def}}{=} \alpha \cdot \overline{g} + (1-\alpha) \cdot \underline{g}$ is the largest possible.
- In our case, $g = -(1-\alpha) \cdot w(x_0)$, so maximizing g simply means selecting the value x_0 for which $w(x_0)$ is the smallest.
- Here, the value $\alpha_- \cdot (x_0 - \underline{x})$ increases with x_0, while the value $\alpha_+ \cdot (\overline{x} - x_0)$ decreases with x_0.

6. Our solution (cont-d)

- Thus, the function $w(x_0)$ – which is the minimum of these two expressions:
  - decreases until the point $\widetilde{x}$ at which these two expressions coincide, and
  - then increases.
- So, the minimum of the worst-case loss $w(x_0)$ is attained at the point $\widetilde{x}$ for which $\alpha_- \cdot (\widetilde{x} - \underline{x}) = \alpha_+ \cdot (\overline{x} - \widetilde{x})$, i.e., for $\widetilde{x} = \widetilde{\alpha} \cdot \overline{x} + (1 - \widetilde{\alpha}) \cdot \underline{x}$.
- Here, we denoted
$$\widetilde{\alpha} \stackrel{\text{def}}{=} \frac{\alpha_+}{\alpha_+ + \alpha_-}.$$
- Interestingly, we get the same expression as with the Hurwicz criterion!

7. References

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