Why Triangular Smoothing?

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1. Formulation of the problem

- Seismic signals $x(t)$ come with a lot of high-frequency noise.
- To decrease the effect of this noise, geophysicists apply *smoothing*, i.e.:
  - replace $x(t)$
  - with $y(t) = \int x(s) \cdot a(t-s)\, ds$ for some smooth function $a(t) \geq 0$ for which $a(-t) = a(t)$ and for $t > 0$, $a(t)$ decreases to 0.
- Empirically, often, the most efficient smoothing comes from using a triangular function

$$a(t) = \max\left(0, 1 - \frac{|t|}{\tau}\right), \text{ for some } \tau > 0.$$

- In this talk, we provide a possible theoretical explanation for this efficiency.

2. Analysis of the problem and the resulting explanation

- From the mathematical viewpoint, smooth means that the function has a derivative.
- However, from the practical viewpoint, if this derivative $a'(t)$ is too high, we could not call this function smooth.
- So, let us interpret smooth as $|a'(t)| \le M$ for some $M > 0$.
- So, the problem takes the following form:
  - on the set of all the functions $a(t)$ for which $a(t) \ge 0$ and $|a'(t)| \le M$ for all t,
  - we need to find a function for which some quantity $E(a)$ – describing efficiency – reaches its maximum.
- According to calculus, in general, the maximum of any function in a region is attained:
  - either at a point where all the derivatives of this functions are 0,
  - or at the border of this region.

3. Analysis of the problem and the resulting explanation (cont-d)

- When the region is small, it is highly improbable that the point at which all derivatives are equal to 0 lies within this region.
- So, with high probability, the maximum is attained at the border of the region.
- When the region is determined by inequalities – as in our case – the border is when one of these inequalities becomes an equality.
- So, we can safely restrict ourselves to the border of the original region.
- To this new – smaller – region, we can apply the same argument.
- We can then conclude that with high probability, the maximum is attained at the border of this new region.
- In other words, the maximum is attained at a point where one more inequality becomes an equality.

4. Analysis of the problem and the resulting explanation (cont-d)

- We can repeat this argument again and again.
- Thus, we conclude that with high probability, the maximum is attained when as many inequalities as possible become equalities.
- In our case, this means that for every t, we have either $a(t) = 0$ or $|a'(t)| = M$.
- For $t > 0$, the function is decreasing.
- So we must have $a'(t) = -M$.
- In this case, we have $a(t) = a(0) - M \cdot t$ – until we reach the value 0.
- In other words, the most effective smoothing is indeed the triangular one.

5. Reference

- S. Greer and S. Fomin, "Matching and merging high-resolution and legacy seismic images", *Geophysics*, 2018, Vol. 83, No. 2, pp. V115–V122.

6. Acknowledgments

This work was supported in part by:

- National Science Foundation grants 1623190, HRD-1834620, HRD-2034030, and EAR-2225395;
- AT&T Fellowship in Information Technology;
- program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and
- a grant from the Hungarian National Research, Development and Innovation Office (NRDI).