# Why Polynomials Accurately Describe the Shape of a Girus

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## 1. Giruses: a brief introduction

- While most viruses are small, there are viruses whose size – 300-500 nanometers – is larger than typical bacteria.
- They are called *giant viruses*, or *giruses*, for short.
- It is important to study giruses because, due to their relatively large size – comparable to the size of bacteria – they play an important role in ecology.
- It is also important to study them since:
  - they are, in many aspects, similar to usual viruses,
  - but, because of their much larger size, it is easier to study their geometric shape.

## 2. What is known about their geometry

- Similar to many other viruses, the surface of a girus consists of several faces.
- So, to describe this surface, it is sufficient to describe the shape of these faces.
- In general, the shape of a surface can be described as $f(x) = 0$ for some function $f(x)$, where $x = (x_1, x_2, x_3)$.
- So, to describe a surface, it is sufficient to describe the corresponding function.
- A usual way to describe a general function is:
  - to select a basis $e_1(x)$, $e_2(x)$, $\ldots$, $e_n(x)$, and
  - to try functions of the type $c_1 \cdot e_1(x) + \ldots + c_n \cdot e_n(x)$.

## 3. What is known about their geometry (cont-d)

- In principle, there can be many different bases.
- It is desirable to select the one for which we can fit the data with the smallest number of parameters $c_i$.
- It turns out that for many marine giruses, a good description of the shape of their faces is provided by cubic polynomials.

## 4. A natural question

- A natural question is: why polynomials – and not any other functions – provide a good basis for this problem?

## 5. A possible explanation of why polynomials are a good approximation

- The surface of each face is smooth.
- So it is reasonable to consider smooth basic functions.
- Every sufficiently smooth function can be extended in Taylor series:

$$e_i(x) = e_{i,0}(x) + e_{i,1}(x) + \ldots$$

- Here, each $e_{i,k}(x)$ is a linear combination of monomials of order $k$, for which $e_{i,k}(\lambda \cdot x) = \lambda^k \cdot e_{i,k}(x)$ for each $\lambda > 0$.
- Giruses vary in size.
- We want to find a basis that fits the shape of several of them.
- So, it is reasonable to select the basis for which:
  - the resulting family $F$ of approximating functions $\sum c_i \cdot e_i(x)$
  - should not change if we re-scale the girus.

## 6. A possible explanation of why polynomials are a good approximation (cont-d)

- In precise terms, for each $\lambda$ and for each $i$, the re-scaled function $e_i(\lambda \cdot x)$ should also belong to $F$.
- Let $e_{i,k}(x)$ be the first non-zero term in the Taylor expansion.
- Then the function $\lambda^{-k} \cdot e_i(\lambda \cdot x) = e_{i,k}(x) + \lambda \cdot e_{i,k+1}(x) + \ldots$ should also be in $F$ for every $\lambda$.
- For $\lambda \to 0$, we conclude that $e_{i,k}(x)$ is in $F$.
- Thus, the difference $e_i(x) - e_{i,k}(x)$ is also in $F$.
- Similar argument shows that:
  - the first term in the Taylor expansion of this difference – which is the second non-zero term in the expansion of $e_i(x)$
  - should also be in $F$.

## 7. A possible explanation of why polynomials are a good approximation (cont-d)

- Similarly, we can prove that each non-zero term $e_{i,j}(x)$ should be in $F$.
- In $F$ we have only $n$ linearly independent functions.
- Polynomials of different order are linearly independent,
- So, we can have only a finite number of different non-zero terms $e_{i,j}(x)$.
- This means that each $e_i(x)$ is a sum of finitely many polynomials – and thus, itself a polynomial.
- This is exactly what we wanted to explain.

## 8. Reference


- M. Mackay, S.-Y. Yi, A. Duval, and C. Xiao, "Geometric modeling of girus trisymmetron", *Abstracts of the 17th Joint UTEP/NMSU Workshop on Mathematics, Computer Science, and Computational Science*, El Paso, Texas, November 7, 2015.

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