More Efficient Computation of the Economic Equilibrium

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1. Formulation of the practical problem

- After each change in the economic situation, market economy eventually settles on some equilibrium state.
- However, often, there are wild oscillations preceding this settling:
  - when the producers either over- or underproduce,
  - which, in both cases, negatively affects their bottom line.
- To avoid such oscillations, it is desirable to set up production levels $q_1, \ldots, q_n$ corresponding to the equilibrium.
- So, it is practically important to compute these equilibrium values.

2. What is known

- There are known formulas for computing the equilibrium.
- Let $D(p)$ denoted the dependence of the demand D on the price p.
- Each producer is supposed to have a hypothesis describing:
  - how the price p will change if this producer changes its production level, i.e.,
  - how the derivative $v_i \stackrel{\text{def}}{=} -\dfrac{\partial p}{\partial q_i}$ depends on q_i.
- Also, for each producer, we know how the production cost depends on the production level q_i.
- In practice, it is sufficient to use a quadratic approximation to this dependence: $b_i \cdot q_i + (1/2) \cdot a_i \cdot q_i^2$.

3. What is known (cont-d)

- Under these assumptions, in the equilibrium, the following equations are satisfied:

$$\sum_{i=1}^{n} q_i = D(p); \quad v_k(q_k) = \frac{1}{\sum_{i\ne k} \dfrac{1}{v_i(q_i)+a_i} - D'(p)} \text{ for all } k.$$

- We thus have $n+1$ equations to determine the $n+1$ unknowns

$$p, q_1, \ldots, q_n.$$

4. Computational challenge and what we do in this talk

- For large n, solving a system of $n+1$ nonlinear equations is computationally complicated.
- In this talk, we show that from the computational viewpoint, we can reduce this problem to solving a single equation with one unknown.
- Solving a single equation is much easier, so this reduction can save a lot of computation time.

5. Our reduction

- If we divide 1 by both sides of second equilibrium equation and add $1/(v_k + a_k)$ to both sides, we conclude that

$$\frac{1}{v_k} + \frac{1}{v_k + a_k} = C, \text{ where } C \stackrel{\text{def}}{=} \sum_i \frac{1}{v_i + a_i} - D'(p).$$

- If we multiply both sides of this equality by $v_k \cdot (v_k + a_k)$, we get a quadratic equation.
- Based on this quadratic equation, we can get an explicit expression for $v_k(q_k)$ in terms of C.
- Now we know how all the values v_i depend on C.
- So, we can use the definition of C to describe how $D'(p)$ depends on C.
- Then, by applying a function which is inverse to $D'(p)$, we can get an expression for p in terms of C.

6. Our reduction (cont-d)

- Similarly, by applying the inverse function v_i^{-1} to $v_i(q_i)$, we can get an expression for each q_i in terms of C.
- Substituting the expressions for q_i and p in terms of C into the first equilibrium equality, we get an equation with one unknown C.
- Once we solve this equation and find C, we can use the explicit expressions for p and q_i in terms of C to compute the desired equilibrium.

7. Reference

- J. G. Flores Muñiz, N. Kalashnykova, V. V. Kalashnikov, and V. Kreinovich, *Public Interest and Public Enterprize: New Developments*, Springer, Cham, Switzerland, 2021.

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